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CALCULUS – EVALUATION TEST 6

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Problem 1. Consider the sequence of real numbers defined by the general term

$$a_n = \frac{n + \sin(\pi n/2)}{3n + 5}, \quad \text{with } n = 1, 2, 3, \dots.$$

Prove whether the sequence is bounded, monotone, and convergent.

Problem 2. Study the convergence of the series

$$\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^2 + 5}, \qquad \sum_{n=1}^{\infty} \frac{\ln(n)}{n}.$$

Problem 3. Determine the exact number of real solutions of the equation $\cos(x) = x$.

Problem 4. Calculate the angle formed by the tangent lines from the right and from the left, at $x_0 = 0$, to the graph of the function

$$f(x) = \begin{cases} \dfrac{\sin(x)}{x}, & \text{if } x < 0, \\[2mm] xe^x + 1, & \text{if } x \geq 0. \end{cases}$$

Problem 5. Find the family of polynomials $P(x)$ such that

$$\lim_{x \to 0} \frac{\sqrt{1 - x^4} - P(x)}{x^7} = 0.$$

Problem 6. Calculate

$$\lim_{x \to 0} \frac{1}{x^3} \int_0^x t^2 \cos(t^2) \, dt \, .$$

Problem 7. Study the convergence of the *improper* integral

$$\int_0^\infty \frac{e^{-x}}{\sqrt{x}}\,dx\,.$$