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## CALCULUS – EVALUATION TEST 7

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**Problem 1.** Consider the sequence of real numbers  $(a_n)_{n \in \mathbb{N}}$  where

$$a_n = \sqrt{n} \frac{2 \cos(\pi(n+1)/2)}{1+n}, \quad \text{with } n = 1, 2, 3, \dots$$

- (a) Study whether the sequence is monotone and bounded.
  - (b) Calculate  $\lim_{n \rightarrow \infty} a_n$ .
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**Problem 2.** Find *all* values of the parameter  $\alpha \in \mathbb{R}$  such that the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{(2\alpha)^{3n}}{7^n \sqrt[3]{n^2 + n}}$$

is convergent.

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**Problem 3.** Approximate the value

$$\ln\left(\frac{3}{2}\right)$$

by a polynomial of suitable degree such that the involved error is smaller than  $10^{-2}$ .

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**Problem 4.** Consider the function

$$f(x) = \begin{cases} \sqrt{1-x} \arctan\left(\frac{1}{x}\right), & \text{if } 0 < x \leq 1, \\ 0, & \text{if } x = 0, \\ \frac{\cos(x) - 1}{x}, & \text{if } x < 0. \end{cases}$$

- (a) Study whether  $f(x)$  is continuous at  $x = 0$ .
- (b) Find the exact number of real solutions of the equation  $f(x) = -1$  in the interval  $(0, 1/2]$ .
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