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CALCULUS – EVALUATION TEST 8

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Problem 1. Solve the following issues.

(a) Consider the *recursive* sequence $(a_n)_{n\in\mathbb{N}}$ defined as

$$a_1 = \frac{1}{2}\,; \qquad a_{n+1} = (a_n)^2 + \frac{4}{25}\,, \quad \text{with } \ n \geq 1\,.$$

Then, prove that $\lim\limits_{n\to\infty} a_n$ exists and calculate its value.

(b) Study the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan\left(n^4\right)}{\sqrt{n^4+1}}$.

Problem 2. Consider

$$f(x) \;=\; \begin{cases} e^{\frac{1}{x}} + \beta\, x\,, & \text{if } \; x < 0\,, \\[2mm] \beta \sin(x) - \dfrac{1}{2}\sin^2(x)\,, & \text{if } \; x \geq 0\,, \end{cases}$$

where $\beta \in \mathbb{R}$ is a parameter.

(a) Find for which values of β the function $f(x)$ is differentiable in $\mathbb{R}$.

(b) Find (if any) the values of β such that the tangent line to the graph of $f(x)$ at $x_0 = 0$ is parallel to the line with equation $y = 3x - 7$.

Problem 3. Approximate the value $\sqrt[3]{1010}$ by means of the Taylor polynomial of degree 3 for the function $f(x) = \sqrt[3]{x}$ about $a = 1000$. Then, find an *upper bound* for the involved error.

Problem 4. Let $F(x) = \displaystyle\int_0^{e^{-x}} \frac{1}{\ln(t)}\, dt$.

(a) Find the global maximum and minimum of $F(x)$ in the interval $x \in [1, 2]$.

(b) Calculate $\displaystyle\lim_{x \to +\infty} x\, F(x)$.

Problem 5. Calculate the following indefinite integrals.

$$\text{(a)} \quad \int x^2\, e^{-3x}\, dx\,. \qquad \text{(b)} \quad \int \frac{x}{x^2 - x + 1}\, dx\,.$$

Problem 6. Consider the *improper* integral $\int_0^{\infty} (x+1)^p \, e^{-x^2} dx$, with $p \in \mathbb{N}$.

(a) Study its convergence for $p = 2$.

(b) Knowing that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, calculate its value for $p = 1$.
