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CALCULUS – EVALUATION TEST 9

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Problem 1. Let $(x_n)_{n\in\mathbb{N}}$ be the *recursive* sequence defined as $x_{n+1} = \sqrt{2x_n + 3}$, for $n \geq 1$, with $x_1 = 1$. Prove that $\lim_{n\to\infty} x_n$ exists and calculate its value.

Problem 2. Study the convergence of the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^3 + n^{-3}}{n^a}$$

depending on the value of the parameter $a \in \mathbb{N}$.

Problem 3. Consider the function

$$f(x) \;=\; \begin{cases} 6\sqrt{x} - x\sqrt{x}\,, & \text{if } \; 0 \le x \le 4\,, \\ (x-4)\, e^{16-x^2} \left[\, 2 - \beta \, \sin\left(\frac{\pi}{8}\, x\right) \right] + 4\,, & \text{if } \; x > 4\,, \end{cases}$$

where $\beta \in \mathbb{R}$ is a parameter.

(a) Find the value of β such that $f(x)$ is differentiable at $x = 4$.

(b) Find the global maximum and minimum of $f(x)$ in the interval $[0, 4]$, if they exist.

Problem 4. Let $F(x) = \int_0^{x^2} \cos(\sqrt{t})\,dt$.

(a) Using the Maclaurin polynomial of degree 2 for $F(x)$, approximate the value

$$\int_0^{0.01} \cos(\sqrt{t})\,dt.$$

(b) Calculate $\lim_{x\to 0} \dfrac{F(x) - x^2}{x^4}$.

Problem 5. Calculate the following indefinite integrals.

$$\text{(a)} \int e^x \cos(2x)\,dx\,. \qquad \text{(b)} \int \frac{2x-3}{x^2+2x+2}\,dx\,.$$

Problem 6. Study for which values of the parameter $k \in \mathbb{R}$ the *improper* integral

$$\int_0^2 \frac{1}{x^k}(1-x)^{k-1}dx$$

is convergent.
