uc3m | Universidad Carlos III de Madrid
Departamento de Matemáticas

DIFFERENTIAL CALCULUS
SELF-EVALUATION I
Degree in Applied Mathematics and Computation

Time: 1 hour

Problem 1 (**0.5 + 1 + 1 + 0.5 = 3 points**)
Compute the following limits (if they exist):

$$a)\ \lim_{x\to\pi/2}(\sin x)^{3/(\cos^2 x)}, \quad b)\ \lim_{t\to 0}\frac{3-5\mathrm{e}^{2/t}}{2+\mathrm{e}^{2/t}},$$

$$c)\ \lim_{x\to 1}\log x\cdot\log(x-1), \quad d)\ \lim_{x\to 0}\frac{1+\sin x-\mathrm{e}^x}{\operatorname{arctg} x}.$$

Problem 2 (**0.5 + 0.5 = 1 point**)
Study the continuity of the functions:

$$a)\ f(x)=\frac{\sqrt{1-\sqrt{9-x^2}}}{x}, \qquad b)\ g(x)=\arcsin(\log|x-1|).$$

Problem 3 (**2 points**)
Prove that the equation

$$2x+\sin\frac{\pi x}{2}=\frac{10}{1+\sqrt{x}}$$

has exactly one root in $[0,\infty)$ and find an interval $[n,n+1)$, with $n\in\mathbb{N}$, where this root is found.

Problem 4 (**2 points**)
Consider a function f such that $f(1/2)=-3$ and $f'(x)=\sqrt{x^2+2}$. If

$$g(x)=x^2 f\big(\frac{x-1}{x}\big),$$

obtain the tangent line to the graph of g at the point $x=2$.

Problem 5 (**2 points**)
Obtain the minimum value of α for which the function $f(x)=|\alpha x^2-2\alpha x+3|$ is differentiable on the whole real line.

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