uc3m | Universidad Carlos III de Madrid
Departamento de Matemáticas

DIFFERENTIAL CALCULUS
SELF-EVALUATION II
Degree in Applied Mathematics and Computation

Time: 90 minutes

Problem 1 (**2 points**)
Plot the graph of the function: $f(x) = x\sqrt{|x^2 - 1|}$.

Problem 2 (**3 points**)

a) Prove that the function $g(x) = \max\{\log(1 + x^2), |x| + \alpha\}$ verifies the hypothesis of the mean value theorem in any interval $[a, b] \in \mathbb{R}$ if and only if $\alpha = \log 2 - 1$.

b) For the previous value of α, obtain the point or points whose existence is guaranteed by the aforementioned theorem applied to the function g in the interval $[-1, 2]$.

c) Obtain the Taylor polynomial of $f(x) = \sin(x/2) + x^2 \mathrm{e}^x$ of order 3 at the origin and estimate the error using that polynomial to approximate the function on $[-1/4, 1/4]$.

Problem 3 (**3 points**)

a) Obtain the limit:

$$\lim_{n\to\infty} \left(\frac{\sqrt[n]{a} + \sqrt[n]{b}}{2} \right)^n, \quad a, b > 0,$$

b) If we have $\lim_{n\to\infty} a_n = \ell$, find: $\displaystyle \lim_{n\to\infty} \frac{a_1 + \frac{a_2}{2} + \cdots + \frac{a_n}{n}}{\log(n+1)}$.

c) Study the convergence of the following recurring sequence and find its limit if it exists:

$$a_0 = \frac{1}{2}, \qquad a_{n+1} = \sqrt{1 + 3a_n} - 1$$

Problem 4 (**1 + 1 = 2 points**)
Study the convergence of the series:

$$a) \sum_{n=2}^{\infty} \frac{2}{(\log n)^{\log n}}, \qquad b) \sum_{n=2}^{\infty} \frac{(-1)^n n! \mathrm{e}^n}{n^{n+1}}.$$

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Elena Romera