

DIFFERENTIAL CALCULUS
CONTROL I

Degree in Applied Mathematics and Computation

Time: 90 minutes

Problem 1 (1 + 1 = 2 points)

Compute the following limits (if they exist):

$$a) \lim_{x \rightarrow 0} (1 + x^2)^{3/(2 \arcsin x)} \quad b) \lim_{x \rightarrow 0} \frac{\sec x - 1}{2x \sin x},$$

Problem 2 (2 points)

Find the values of λ for which the function: $f(x) = \frac{1}{\lambda x^2 - 4\lambda x + 4}$ is continuous on \mathbb{R} .

Problem 3 (2 points)

Prove that a polynomial of even degree:

$$f(x) = a_{2n}x^{2n} + a_{2n-1}x^{2n-1} + \dots + a_1x + a_0, \quad a_{2n} \neq 0,$$

is bounded below if $a_{2n} > 0$ and it is bounded above if $a_{2n} < 0$.

Problem 4 (2 points)

The equation

$$\begin{cases} e^{-f} f' = 2 - \log(x + 1), \\ f(0) = 2, \end{cases}$$

defines a differentiable one-to-one (bijective) function f on the interval $(-1, 1)$. We define the function $g(x) = f^{-1}(x + 2)$. Obtain the limit

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-\sin x}}{g(x)}.$$

Problem 5 (2 points)

Find the absolute maxima and minima of the function $f(x) = 2x^{5/3} + 5x^{2/3}$ on the interval $[-2, 1]$.

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