uc3m | Universidad Carlos III de Madrid
Departamento de Matemáticas

DIFFERENTIAL CALCULUS
CONTROL II
Degree in Applied Mathematics and Computation

Time: 90 minutes

Problem 1 (**2,5 points**)

Obtain the graphic representation of: $y = \dfrac{\mathrm{e}^{-x}}{x^2-1}$. Deduce the convexity and concavity without the second derivative.

Problem 2 (**1 + 1 = 2 points**)

a) Use Taylor's theorem to compute: (using other method it is worth 0.8 points)

$$\lim_{x\to 0} \frac{1}{x}\left(\frac{1}{x} - \cot x\right).$$

b) Obtain the Taylor polynomial (in its general form) of the functions $f(x) = \log(1-x)$ and $g(x) = \log(1-x^2)$ at $x_0 = 0$.

Problem 3 (**1 + 1,5 = 2.5 points**)

a) Compute the limit: $\lim_{n\to\infty} (\sqrt[4]{n^2+1} - \sqrt{n+1})$.

b) Study the convergence of the sequence defined by: $a_n = \sqrt{3+2a_{n-1}}, \qquad a_0 = 0.$

Problem 4 (**1 + 1 + 1 = 3 points**)

a) Study the convergence (conditional and absolute) of the series: $\displaystyle\sum_{n=0}^{\infty} \frac{(-4)^n}{\mathrm{e}^n n!}$.

b) Study the convergence interval and the sum of the series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}.$$

c) Obtain the interval of convergence of the series: $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}(x-2)^n$.

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