

DIFFERENTIAL CALCULUS  
EXTRAORDINARY EXAM  
Degree in Applied Mathematics and Computation

Time: 3 hours

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Problem 1 (1 + 1 + 1 = 3 points)

- a) Calculate the domain of the function  $f(x) = \arcsin\left(\frac{x}{x^2+1}\right)$ ,
- b) Find the minimum value  $k$  such that  $f$  is injective on  $[k, \infty)$  and obtain the inverse function on that interval.
- c) Prove that

$$\arctan\left(\frac{1+x}{1-x}\right) - \arctan(x) = \frac{\pi}{4}, \quad x < 1.$$

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Problem 2 (2 points)

Plot the function  $f(x) = x\sqrt{|x^2-4|}$ , with all the calculations.

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Problem 3 (1 + 1 = 2 points)

a) Compute the limit:  $\lim_{n \rightarrow \infty} \frac{\sum_{k=1}^n \sin\left(\frac{\pi}{k}\right)}{\log n}$ .

- b) Study the convergence of the sequence defined by:  $a_0 = 1/2$ ,  $a_{n+1} = 2 + \frac{4}{a_n}$ .

*Hint:* Observe that  $a_n > 2$  for  $n \geq 1$ .

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Problem 4 (1 + 1 + 1 = 3 points)

- a) Study the convergence of the series  $\sum_{n=1}^{\infty} \frac{n(1+a)^n}{e^{an}}$  for different values of  $a > -1$ .

- b) Sum and obtain the interval of convergence of the series  $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2^n (2n+1)!}$ .

- c) Obtain the Taylor series and the interval of convergence of  $f(x) = \log\left(\frac{1}{1+2x}\right) + 2x$ .
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Open Course Ware, UC3M

Elena Romera

