Chapter 2 : Exercises

Exercise 2.1 The same input signal is applied to 4 analog modulators different. The frequency response at the output of the four modulators, giving rise to the four spectrums represented in Figure 2.1. have note that the abscissa represents frequency in kHz, and that each subfigure has different scales.

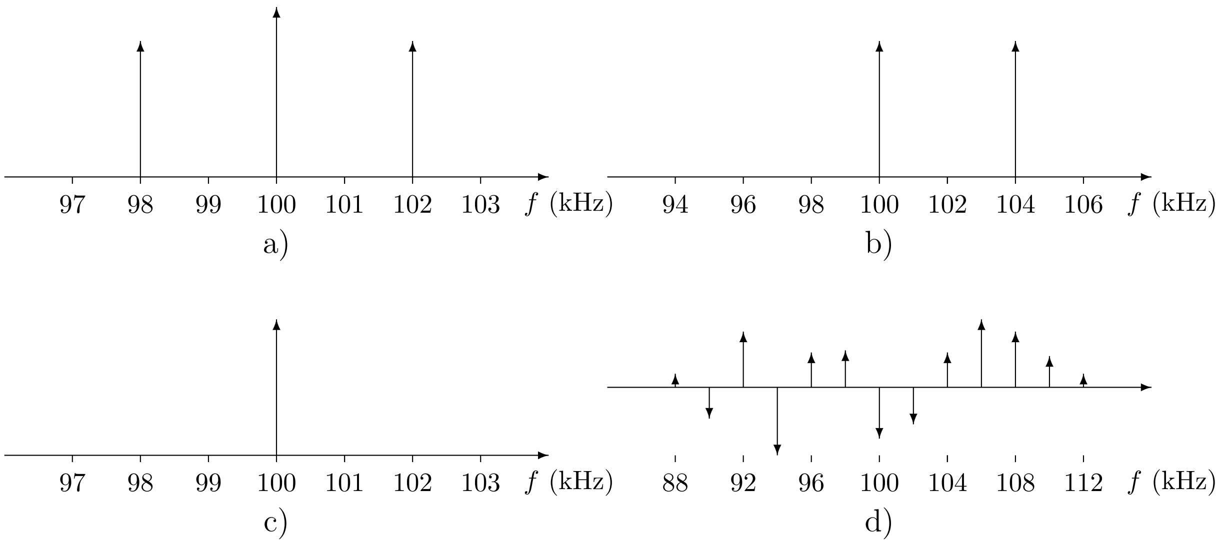


Figure 2.1: Frequency responses of Exercise 2.1.

a) Specify which of the possible variants of analog modulations each one of the spectra belongs to, and which is the frequency of the carrier signal of each modulator.

b) Provide the complete analytic expression (including the possible numerical values that can be extracted in view of the figures) of the input signal to the modulators (modulating signal $m(t)$).

Exercise 2.2 In an FM modulator with modulation index 5 and carrier frequency $f_c = 100$ MHz, a modulating signal $m(t) = \cos(2\pi f_m t)$ is introduced, with $f_m = 2$ MHz. Explain what the spectrum of the signal is like and draw it, clearly indicating the frequencies and amplitudes of the most relevant components.

Exercise 2.3 The modulating signal of an analog communications system, $m(t)$, is a signal with bandwidth B Hz (or $W = 2\pi B$ rad/s) with power spectral density

$$S_M(j\omega) = \begin{cases} \frac{|\omega|}{W}, & \text{si } |\omega| \leq W = 2\pi B \\ 0, & \text{si } |\omega| > W = 2\pi B \end{cases}.$$

The carrier has a frequency $f_c >> B$ Hz.

a) Indicate the bandwidth of the signal modulated by conventional AM with modulation index a, and draw its power spectral density, $S_S(j\omega)$.

b) Indicate the bandwidth of the signal modulated by double-sideband amplitude modulation and draw its power spectral density, $S_S(j\omega)$.

c) Calculate the bandwidth of the signal modulated by single-sideband amplitude modulation and plot its power spectral density, $S_S(j\omega)$, for the lower-sideband case.

d) Calculate the bandwidth of the modulated signal by means of an FM modulation with modulation index $\beta = 5$.

Exercise 2.4 The modulating signal of an analog communications system is $m(t) = \cos(2\pi f_m t)$, with $f_m = 2$ MHz. The carrier $c(t) = A_c \cos(\omega_c t)$, where $\omega_c = 2\pi f_c$ with a frequency $f_c = 100$ MHz.

a) Draw the power spectral density and indicate the power of the modulated signal $s(t)$ if an amplitude modulation is used with modulation index $a = 0.5$.

b) Draw the power spectral density and indicate the power of the modulated signal $s(t)$ if a double sideband amplitude modulation is used. side band.

c) Draw the power spectral density of the modulated signal $s(t)$ if single-sideband amplitude modulation is used (lower side band).

Exercise 2.5 In an analog communications system, the signal to be transmitted (modulating signal) is a baseband signal with bandwidth B Hz.

a) Give the spectral efficiency (giving the bandwidth) of a conventional AM modulation (with carrier), and say how it behaves against noise compared to a baseband transmission (without modulating).

b) Give the spectral efficiency (giving the bandwidth) of a carrierless double-sideband AM modulation, and explain how it behaves in front of the noise comparing it with an unmodulated transmission.

c) Give the spectral efficiency (giving the bandwidth) of a single-sideband AM modulation, and tell how it behaves in front of the noise comparing it with an unmodulated transmission.

d) Give the spectral efficiency (giving the bandwidth) of a vestigial sideband AM modulation, and say how it behaves in front of the noise comparing it with an unmodulated transmission.

e) Give the spectral efficiency (giving the bandwidth) of an FM modulation with modulation index $\beta = 3$, and explain how it behaves in front of the noise comparing it with an unmodulated transmission.

Exercise 2.6 The following equations present the analytical expressions of the modulated signal $s(t)$ for a modulating signal $m(t)$ in different analog modulations.

a) $s_1(t) = A_c\, m(t)\, \cos(\omega_c t + \phi_c)$

b) $s_2(t) = A_c\, \cos(\omega_c t + a\, m(t))$

c) $s_3(t) = A_c\, [1 + m(t)]\, \cos(\omega_c t + \phi_c)$

d) $s_4(t) = A_c\, m(t)\, \cos(\omega_c t + \phi_c) - A_c\, \hat{m}(t)\, \sin(\omega_c t + \phi_c)$

e) $s_5(t) = A_c\, \cos(\omega_c t + b \int_{-\infty}^{t} m(\tau)\, d\tau)$

f) $s_6(t) = A_c\, m(t)\, \cos(\omega_c t + \phi_c) + A_c\, \hat{m}(t)\, \sin(\omega_c t + \phi_c)$

where $\hat{m}(t)$ denotes the Hilbert transform of the modulating signal. Identify each modulation.

Exercise 2.7 Identify the variant of the analog modulation for each one of the following analytical expressions of the spectral density of power, $S_S(j\omega)$, or Fourier transform, $S(j\omega)$, of the modulated signal $s(t)$.

a) $S_S(j\omega) = \begin{cases} 0, & |\omega| > \omega_c \\ A_c^2\, [S_M(j\omega - j\omega_c) + S_M(j\omega + j\omega_c)], & |\omega| \leq \omega_c \end{cases}$

b) $S_S(j\omega) = \frac{A_c^2}{4}\, [S_M(j\omega - j\omega_c) + S_M(j\omega + j\omega_c)]$

c) $S(j\omega) = A_c\, \pi\, [\delta(\omega - \omega_c)\, e^{j\phi_c} + \delta(\omega + \omega_c)\, e^{-j\phi_c}] + \frac{A_c}{2}\, [M_a(j\omega - j\omega_c)\, e^{j\phi_c} + M_a(j\omega + j\omega_c)\, e^{-j\phi_c}]$

d) $S_S(j\omega) = \begin{cases} A_c^2\, [S_M(j\omega - j\omega_c) + S_M(j\omega + j\omega_c)], & |\omega| \geq \omega_c \\ 0, & |\omega| < \omega_c \end{cases}$

e) $S(j\omega) = \sum_{n=-\infty}^{\infty} A_c\, \pi\, J_n(3)\, [\delta(\omega - \omega_c - n\,\omega_m) + \delta(\omega + \omega_c + n\,\omega_m)]$

f) $S_S(j\omega) = \frac{A_c^2}{2}\, \pi\, [\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + \frac{A_c^2}{4}\, [S_{M_a}(j\omega - j\omega_c) + S_{M_a}(j\omega + j\omega_c)]$

Exercise 2.8 For amplitude modulations (AM), and in particular for the receivers

a) Explain what a coherent receiver is and draw a block diagram with its functional blocks.

b) Say which are the variants of AM that require the use of a coherent receiver and which can use a simpler receiver (in this case say which simple receiver can be used).

c) Indicate, specifically for each of the AM variants that require their use, what is the effect of not using a coherent receiver.

Exercise 2.9 The modulating signal of a communications system is modeled by a stationary random process with zero mean and whose power spectral density is the one shown in Figure 2.2.

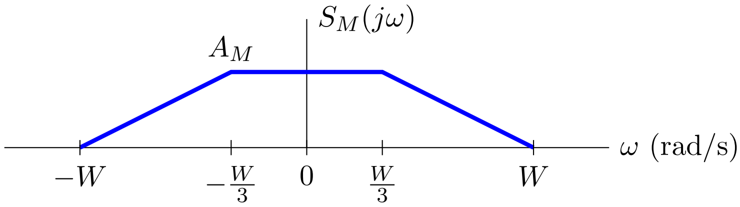


Figure 2.2: Power spectral density of the modulating singal for Exercise 2.9.

The communications system uses single sideband (SSB) modulation with lower sideband and carrier frequency $\omega_c >> W$ rad/s.

a) Represents the power spectral density of the modulated signal.

b) Compare, in terms of spectral efficiency, SSB modulation with conventional AM, double sideband AM, and FM modulation with modulation index $\beta > 1$.

Exercise 2.10 In an analog transmission system, whose carrier frequency is 50 kHz and amplitude $A_c = 1$, it will be transmitted as a signal modulating a 5 kHz tone, i.e.

$$m(t) = \cos(2\pi f_m t), \text{ con } f_m = 5 \times 10^3 \text{ Hz}.$$

a) If conventional AM modulation is used, calculate the bandwidth of the modulated signal, and approximately plot the modulated signal in the time domain if the modulation index is $a = \frac{1}{2}$ (label both axes appropriately, especially the amplitude axes of the modulated signal).

b) If an amplitude modulation is considered, but it is desired to eliminate the inefficiency that carrier transmission supposes, what modulations would be possible? Indicate the bandwidth of the modulated signal in all possible cases.

c) In the case of using FM modulation with an index of modulation $\beta_f = 7$, indicate what is the theoretical bandwidth (exactly), define what is the effective bandwidth of the modulated signal, and say what its value would be for this case.

d) When comparing the angular modulations against the linear or amplitude modulations, explain the fundamental advantage and the main disadvantage of the former with respect to the latter.

Exercise 2.11 Figure 2.3 plots four modulated signal.

a) Figures A, B, C and D show the modulated signal when the same modulating signal, $m(t)$, is transmitted using four types of analog modulations: conventional AM (in this case with modulation index $a = \frac{1}{2}$), double sideband modulation (DBL), phase modulation (PM) and frequency modulation (FM). Identify the type of modulation each of the figures corresponds to, indicating its distinctive features (without this indication, the answer will not be considered valid).
<u>REMARK</u>: to facilitate identification, the carrier $c(t)$ has been included in the 4 figures (thin red line).

b) Say what type of analog modulation, among all the possible ones (do not limit yourself to the 4 indicated in the previous section), you would use in the cases that are going to be described below. For each case, you must indicate precisely what characteristic of the modulation makes it appropriate, and what advantage it provides over other modulations.

   i) System that transmits high-fidelity signals, where the most important thing is the quality of the received signal.

   ii) System in which the main priority is that the receivers are as economical as possible.

   iii) A voice signal multiplexing system over the same cable, with a 400 MHz bandwidth, in which it is desired to be able to multiplex the greatest possible number of voice signals.

Exercise 2.12 For a vestigial sideband amplitude modulation, with a modulating signal of bandwidth W rad/s

a) Explain how the modulated signal is generated, and draw a block diagram with the different elements that make up the transmitter.

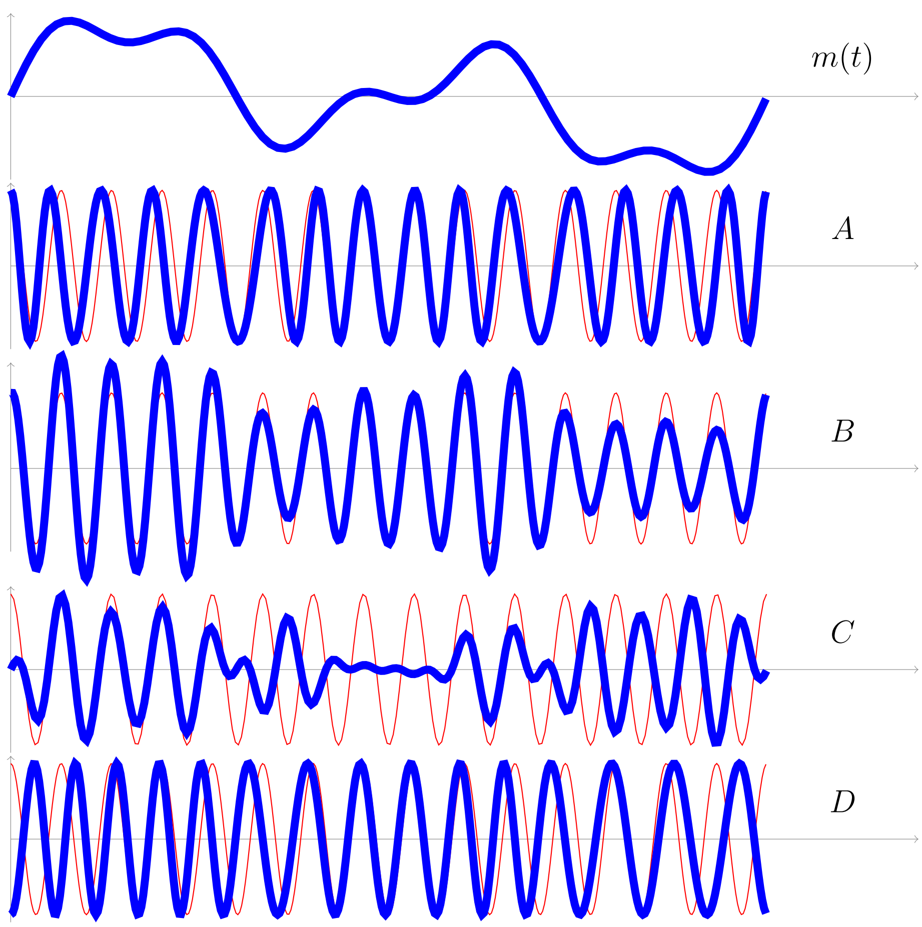


Figure 2.3: Signals for Exercise 2.11.

b) Indicate what condition the vestigial sideband filter has to meet for this type of modulation, and for each of the filters in Figure 2.4, show whether or not this condition is met, and if so, say whether it would be used for upper or lower sideband modulation.

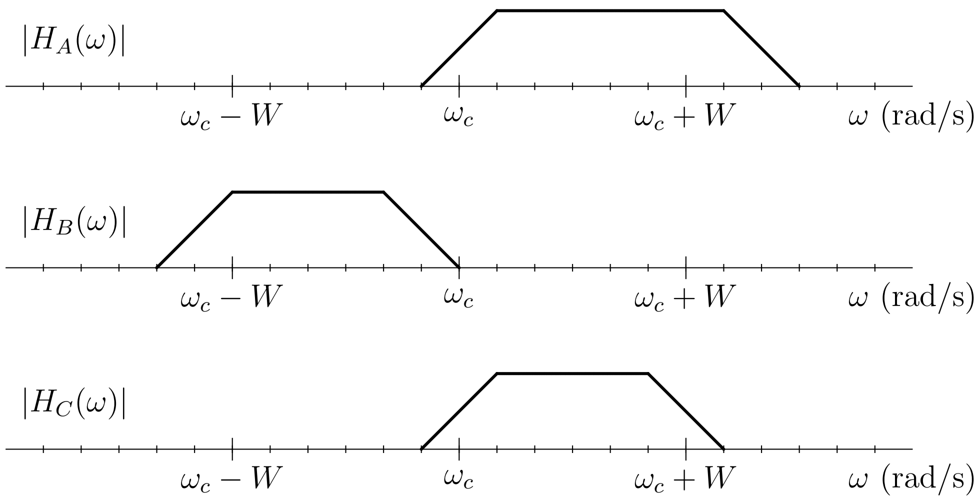


Figure 2.4: Frequency reponses of the filters of Exercise 2.12.

Exercise 2.13 An analog communications system is designed to transmit a modulating signal that has the following power spectral density

$$S_M(j\omega) = \begin{cases} A_M\left(1 - \cos\left(2\pi\frac{\omega}{W}\right)\right), & \text{if } |\omega| \leq W \\ 0, & \text{in other case} \end{cases}$$

where $A_M = 2 \times 10^{-18}$, $W = 2\pi B$ is the bandwidth in rad/s, with $B = 5$ kHz being the bandwidth in Hz. For simplicity, it is assumed that in the transmission the modulated signal does not suffer any attenuation, only thermal noise is added with the usual statistical model, and that the system works at a noise temperature of 290 Kelvin degrees.

a) Plot the power spectral density of the modulating signal, and calculate the signal-to-noise ratio at the output of the receiver if the baseband signal is transmitted without modulating and the receiver is an ideal low-pass filter with bandwidth B Hz.

b) A double sideband modulation with a carrier frequency $f_c = 100$ kHz is now used. For simplicity, consider the carrier amplitude to be $A_c = 1$.

   i) Represent the power spectral density of the modulated signal and calculate the bandwidth of this signal

   ii) If a coherent receiver with a previous noise filter is used in the receiver, and this filter is an ideal filter, draw the frequency response of the optimal noise filter for this system (the one that maximizes the signal-to-noise ratio) and calculate the signal-to-noise ratio, in dB, at the output of the coherent receiver.

c) Repeat the previous section for an upper sideband single sideband modulation

Exercise 2.14 Various analog communication systems designed to transmit a modulating signal having the following power spectral density are to be considered.

$$S_M(j\omega) = \begin{cases} A_M\left(1 - \left(\frac{\omega}{W}\right)^2\right), & \text{if } |\omega| \leq W \\ 0, & \text{in other case} \end{cases}$$

where $W = 2\pi B$ and $B = 5$ kHz is the bandwidth of the modulating signal.In all modulations to be considered, the carrier frequency is $f_c = 100$ kHz. For simplicity, consider that the range of the modulating signal is $-1 \leq m(t) \leq +1$, and that the amplitude of the carrier is $A_c = 1$.

a) Represent the power spectral density of the modulated signal for a conventional AM amplitude modulation with modulation index $a = \frac{1}{2}$, and indicate the bandwidth of said modulated signal in Hz.

b) Represent the power spectral density of the modulated signal for a double-sideband amplitude modulation, and indicate the bandwidth of said modulated signal in Hz.

c) Plot the power spectral density of the modulated signal for single-sideband, lower-sideband amplitude modulation, and indicate the bandwidth of that modulated signal in Hz.

d) If a vestigial sideband modulation is used, with upper sideband, with an excess (vestige) bandwidth of 500 Hz

   i) Represent the frequency response of a vestigial sideband filter that meets the conditions required in said modulation.

   ii) Plot the frequency response of the modulated signal using the above vestigial sideband filter.

   iii) Indicate the bandwidth in Hz of the modulated signal.

e) In this case an angular modulation is considered

   i) If the modulation is narrowband phase modulation, approximately represent the power spectral density of the modulated signal and indicate the bandwidth of the modulated signal.

   ii) If the modulation is a frequency modulation with modulation index $\beta = 5$, indicate the bandwidth of the modulated signal.

Exercise 2.15 Different analog modulations must be identified, and the characteristics of the modulated signal described.

a) Figure 2.5 represents the power spectral density of the modulated signal with 4 different analog modulations, when the modulating signal is the same for the 4 modulations. Plot the power spectral density of the modulating signal and indicate its bandwidth, and for each of the figures, identify the modulation precisely (including its full name and the corresponding variant in case that type of modulation has different variants), describing its distinctive features, and provide the carrier frequency.

b) Figure 2.6 shows the modulated signals for 3 analog modulations, when the modulating signal is the same for all of them (carrier has been included in red thin line). Plot the modulating signal, and for each of the figures, identify the modulation precisely (including its full name and the corresponding variant in case that type of modulation has different variants), describing its distinctive features.

Exercise 2.16 The modulating signal to be used as input to several analog modulators has the power spectral density of Figure 2.7. The amplitude of the carrier will be $A_c = 1$ and its frequency $f_c = 1$ MHz, for all the modulations. For simplicity, assume from now on that the range of the modulating signal is $c_M = 1$, and that its power is $P_M = 1$ Watts. Transmitting the signal in base

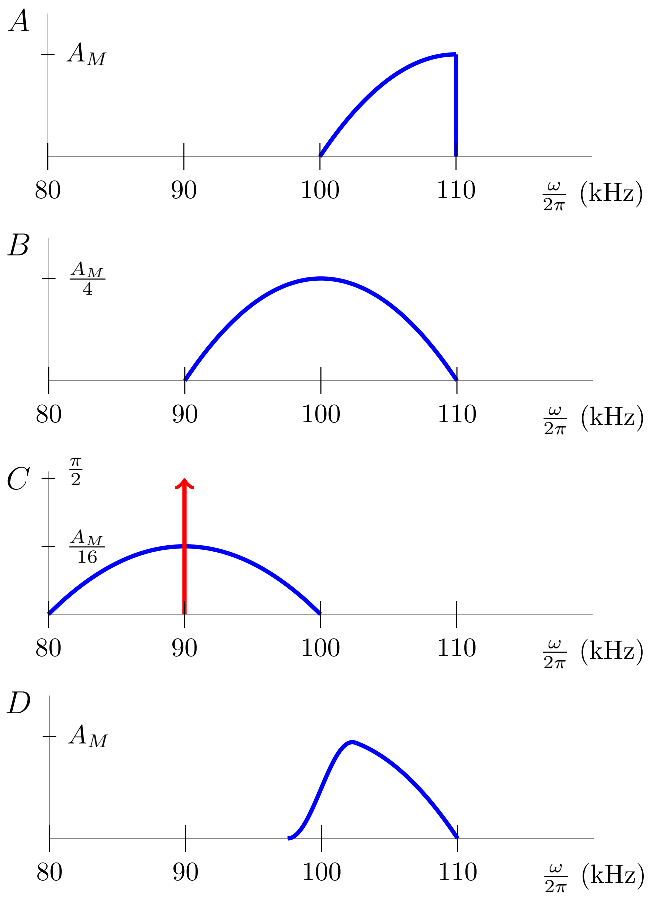


Figure 2.5: Power spectral density $S_X(j\omega)$ of the 4 modulated signals for Exercise 2.15.

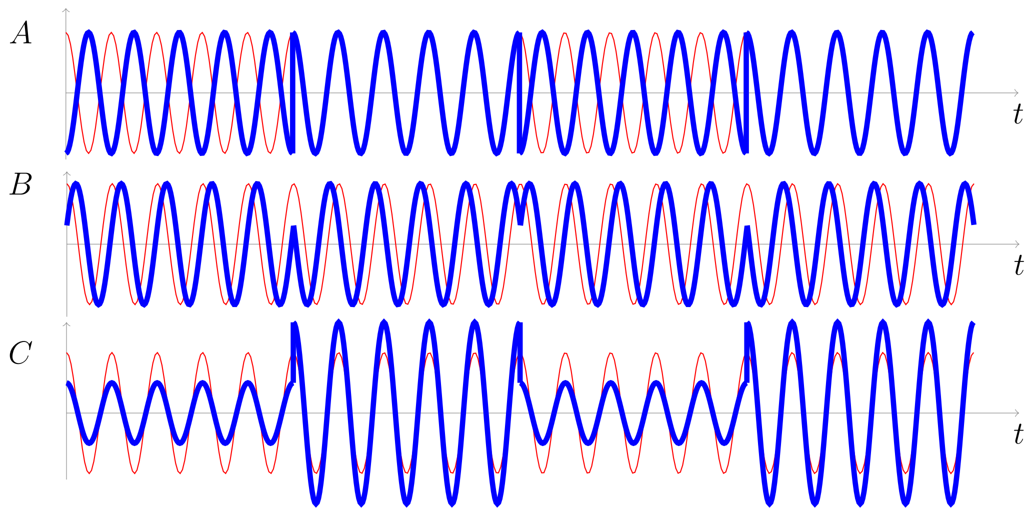


Figure 2.6: Modulated signals $s(t)$ for Exercise 2.15.

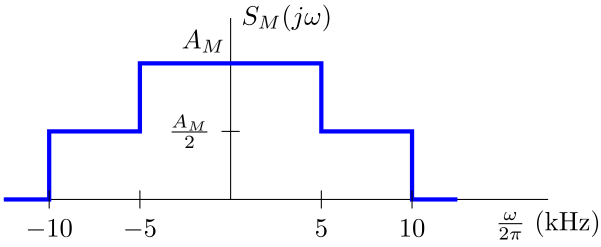


Figure 2.7: Power spectral density for the modulating signal of Exercise 2.16.

band, there is a signal-to-noise ratio at the output of the receiver of 20 dB. Assume that for all modulations, the received power at the receiver input is equal to the received power transmitting at baseband.

a) If a conventional AM modulation is used, with modulation index $a = \frac{1}{2}$, represent the power spectral density of the modulated signal, indicate the bandwidth in Hz of said signal, and the signal-to-noise ratio in dB of the signal received with a coherent receiver.

b) If double-sideband AM modulation is used, represent the power spectral density of the modulated signal, indicate the bandwidth in Hz of said signal, and the signal-to-noise ratio in dB of the signal received with a coherent receiver.

c) If single-sideband AM modulation is used, with upper sideband, represent the power spectral density of the modulated signal, indicate the bandwidth in Hz of said signal, and the signal-to-noise ratio in dB of the signal received with a coherent receiver.

d) If a vestigial sideband AM modulation is used, with lower sideband and a vestige of $\Delta_B = 1$ kHz, represent the frequency response of a valid vestigial sideband filter for this type of modulation, indicate the bandwidth in Hz of the modulated signal, and the signal-to-noise ratio in dB of the signal received with a coherent receiver.

e) Indicate the bandwidth in Hz of the modulated signal, and the signal-to-noise ratio in dB of the received signal, if the following modulations are used:

   i) Frequency modulation with modulation index $\beta = 3$.

   ii) Phase modulation with modulation index $\beta = 5$.

Exercise 2.17 The power spectral density of the modulator of an analog communications system is

$$S_M(j\omega) = \begin{cases} 10^{-18}\left[1+\cos\left(\frac{\omega}{2\times 10^5}\right)\right], & \text{if } |\omega| \leq 2\pi\times 10^5 \text{ rad/s} \\ 0, & \text{in other case} \end{cases}$$

The carrier has a frequency of 1 MHz and a unit amplitude. It is assumed that the modulated signal is transmitted to the receiver in an ideal way, without attenuation, and with the addition of thermal noise, assuming a noise temperature of 290º Kelvin, as the only distortion.

a) Draw the block diagram of a coherent receiver for amplitude modulations, including the noise filter, explain the fundamental condition that the receiver must meet to be coherent, and represent the ideal frequency response of the low-pass filter included in said receiver.

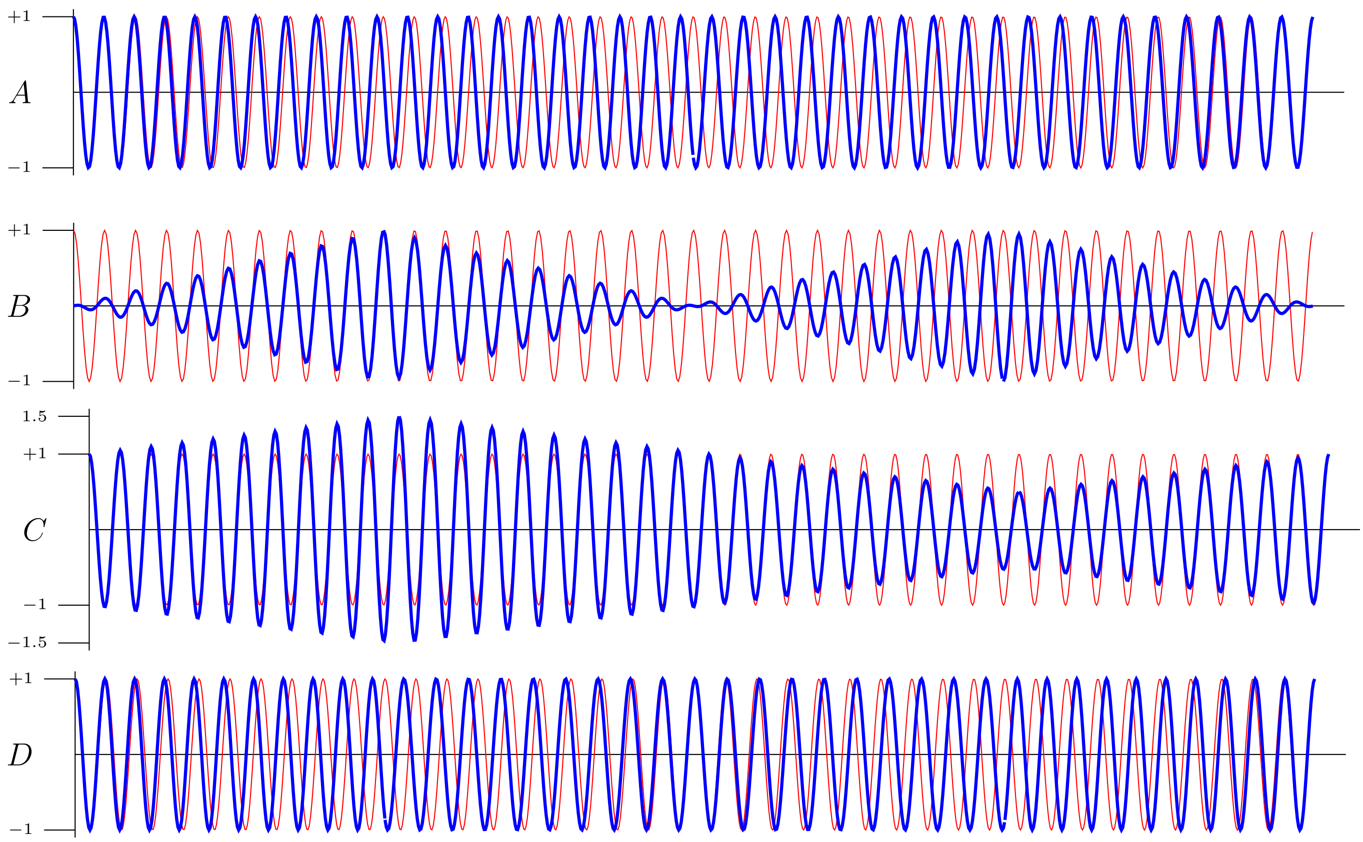
b) For a double-sideband amplitude modulation, indicate the bandwidth in Hz of the modulated signal, draw the frequency response of the optimal noise filter of the coherent receiver, and calculate the signal-to-noise ratio in dB obtained with said receiver.

c) For double sideband modulation, if a non-coherent receiver is used, in which the phase difference between the transmitter and receiver carriers is 45 degrees, calculate the signal-to-noise ratio in dB at the output of said receiver.

d) For a vestigial sideband modulation with a 10 kHz trace, lower sideband, calculate the bandwidth in Hz of the modulated signal, the signal-to-noise ratio at the output of a coherent receiver, and plot the frequency response of the optimal noise filter for the coherent receiver of that modulation.

e) For a single-sideband modulation, with upper sideband, represent the power spectral density of the modulated signal, and calculate the bandwidth in Hz of said signal, as well as the signal-to-noise ratio at the output of a coherent receiver for this modulation.

Exercise 2.18 Figures A, B, C and D show the modulated signal when the same modulating signal $m(t)$ is transmitted using four types of analog modulations: conventional AM with modulation index $a = 0.5$, double sideband modulation (DSB), phase modulation (PM) and frequency modulation (FM). The carrier has been included on the 4 figures (thin red line).



a) Plot the modulating signal $m(t)$, which is common to all four modulations.

b) Identify the type of modulation each of the figures corresponds to, indicating its distinctive features (without this indication, the answer will not be considered valid).

Exercise 2.19 The modulating signal of an analog communications system has the following power spectral density

$$S_M(j\omega) = \begin{cases} \frac{A_M}{2} + \frac{A_M}{2}\,\left(\frac{\omega}{4\pi\times 10^6}\right)^2, & \text{if } |\omega| \leq 4\pi\times 10^6 \text{ rad/s} \\ 0 & \text{in other case} \end{cases}$$

a dynamic range $-2 \leq m(t) \leq 2$ and a power of 1 Watt. It will be modulated with several types of modulation, in all cases with a carrier of amplitude 1 and frequency 100 MHz. In transmission, the noise temperature is 290 Kelvin degrees and the power of the modulated signal that reaches the input of the receiver is 8 pW.

a) For single sideband (lower sideband) amplitude modulation:

  i) Draw the block diagram of the Hartley modulator for this modulation.
  ii) Plot the power spectral density of the modulated signal.
  iii) Calculate the bandwidth in Hz of the modulated signal.
  iv) Calculate the signal-to-noise ratio in dB at the output of a coherent receiver.

b) For a vestigial sideband (upper sideband) amplitude modulation, with a vestige of 500 kHz:

  i) Plot the frequency response of a valid vestigial sideband filter, and explain the conditions that such a filter must satisfy.
  ii) Plot the power spectral density of the modulated signal.
  iii) Calculate the bandwidth in Hz of the modulated signal.
  iv) Calculate the signal-to-noise ratio in dB at the output of a coherent receiver.

c) For a frequency modulation, with modulation index 5, calculate:

  i) The bandwidth in Hz of the modulated signal.
  ii) The signal-to-noise ratio in dB at the output of the receiver.