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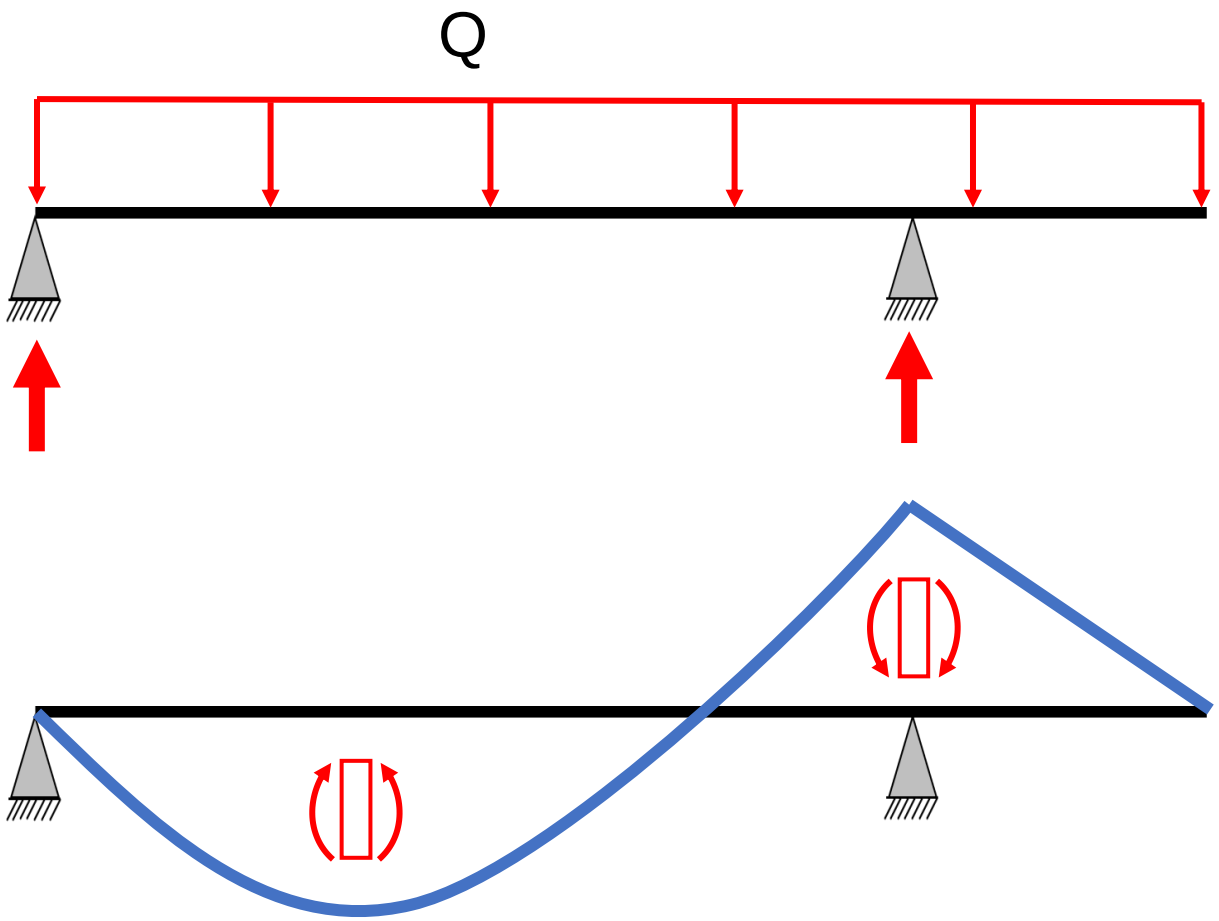
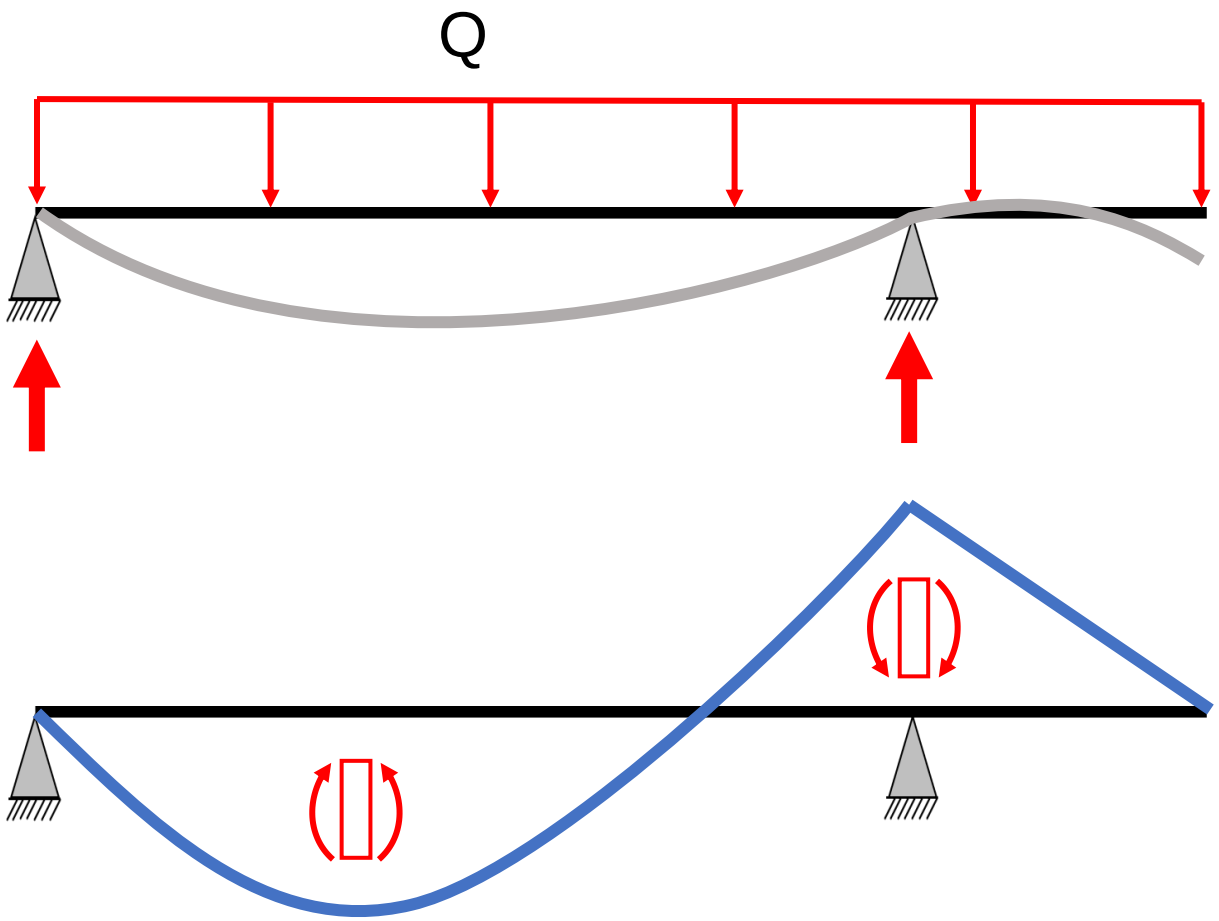
Teoría de Estructuras y Construcciones Industriales

Carlos Santiuste Romero, Sara Garzón Hernández, Liu Jiao Wang,

Manuel Cuadrado Sanguino, Luis Jiménez Girón, Daniel Herrero Adán

Teoremas de Castigliano para desplazamientos

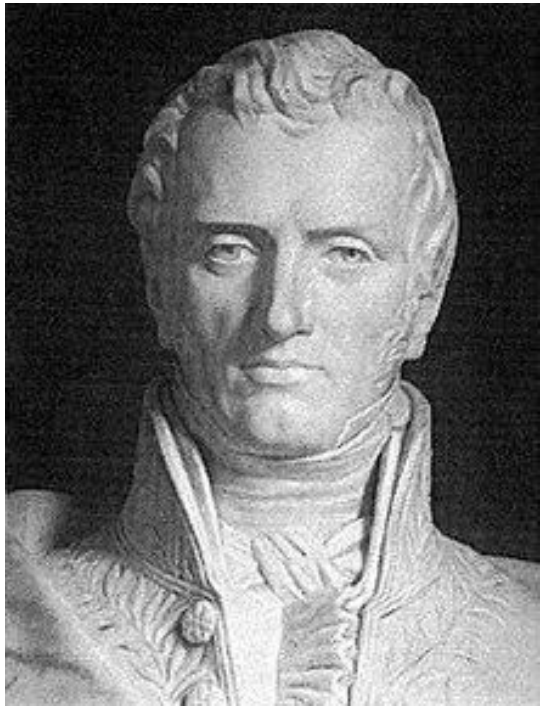

Q


Q

Ecuaciones de NAVIER-BRESSE

$$\theta_B = \theta_A + \int_A^B \frac{M}{EI} dx$$

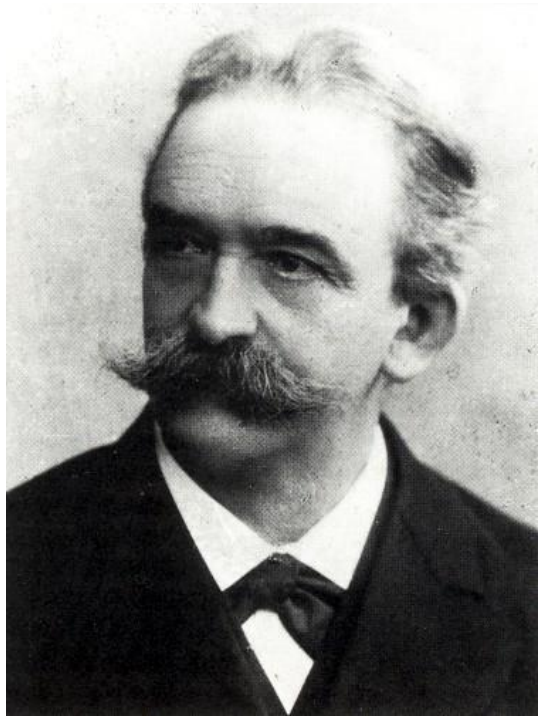
$$v_B = v_A + \theta_A (x_B - x_A) + \int_A^B \frac{M}{EI} (x_B - x) dx$$

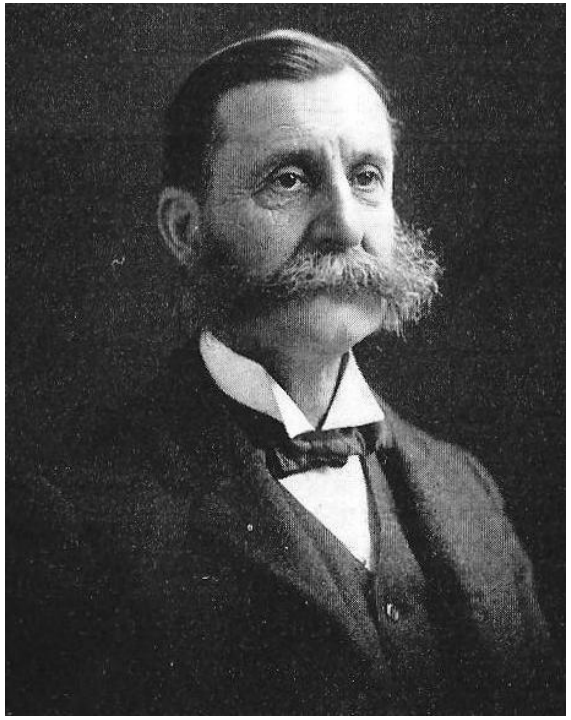


Teoremas de Mohr

$$\theta_{AB} = \theta_B - \theta_A = \int_A^B \frac{M_f(x)}{EI} dx$$

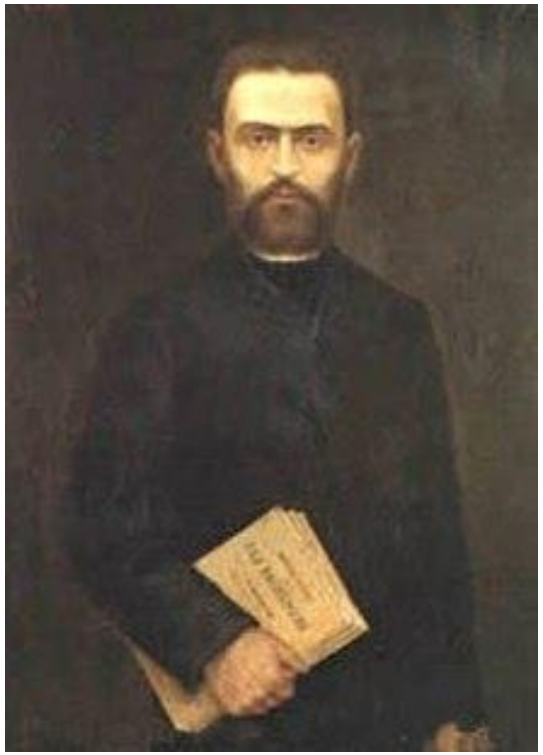
$$\Delta_{B\|A} = \int_A^B \frac{M_f(x)}{EI} (x_B - x) dx$$



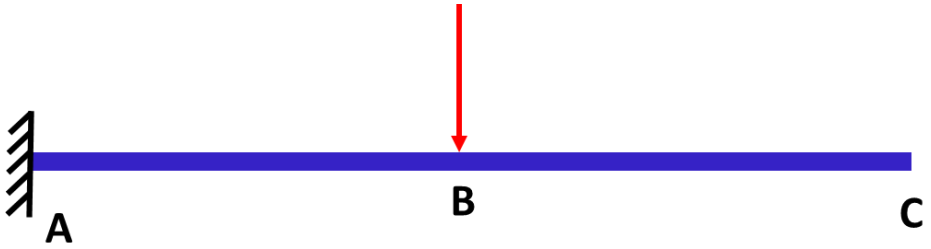


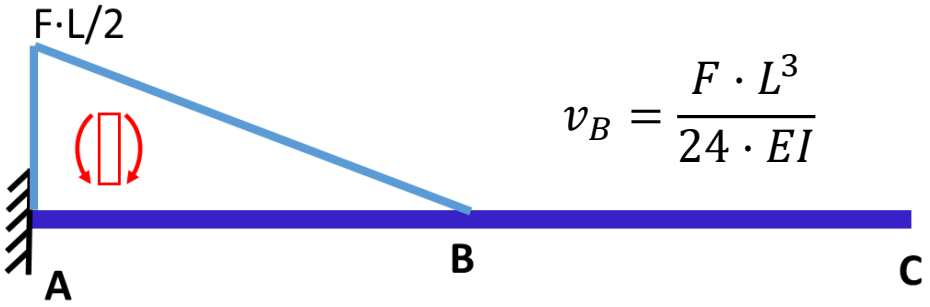
Carlo Alberto CASTIGLIANO

1847-1884



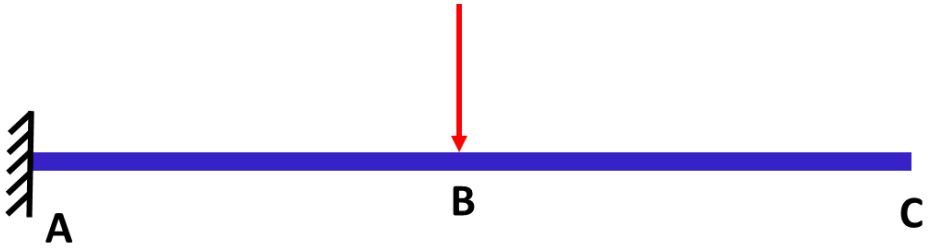
Energía potencial

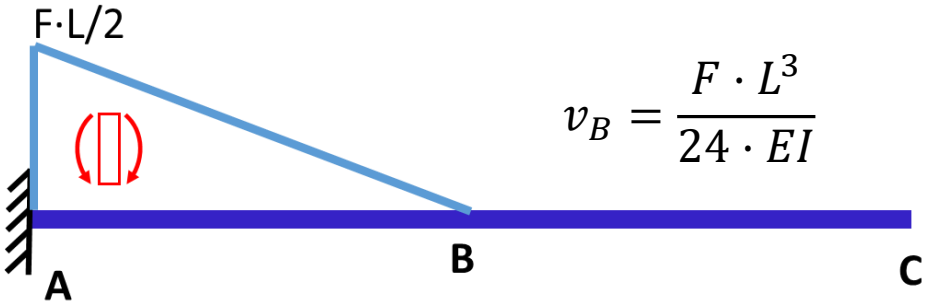




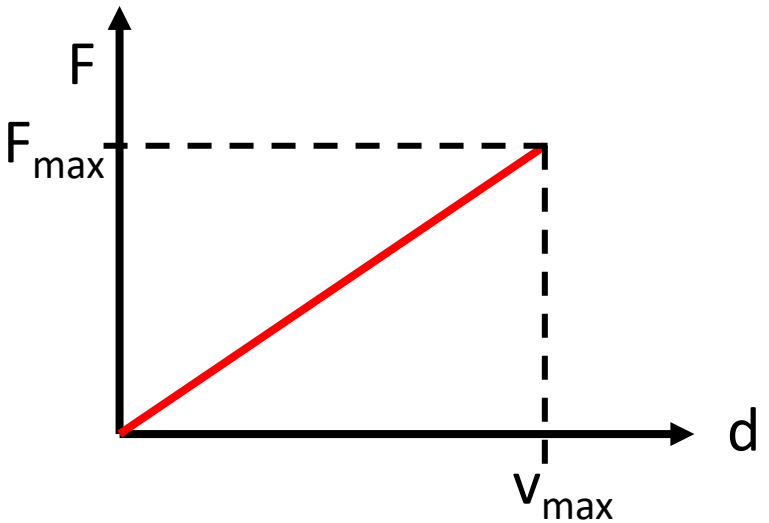
$$v_B = \frac{F \cdot L^3}{24 \cdot EI}$$

Energía potencial

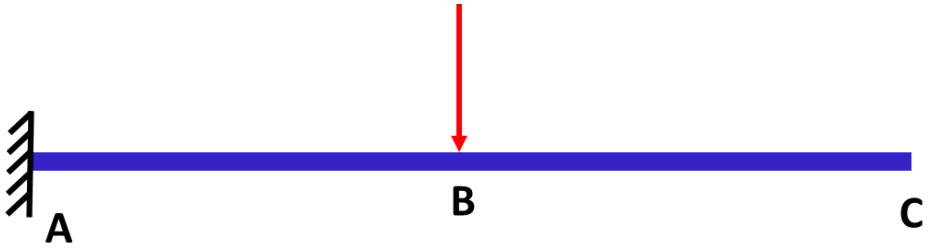


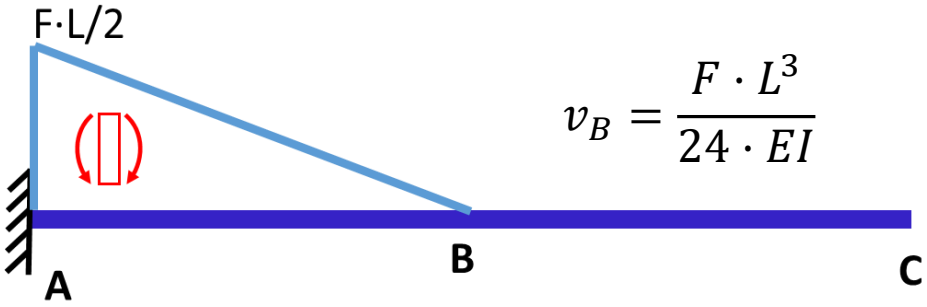


$$v_B = \frac{F \cdot L^3}{24 \cdot EI}$$

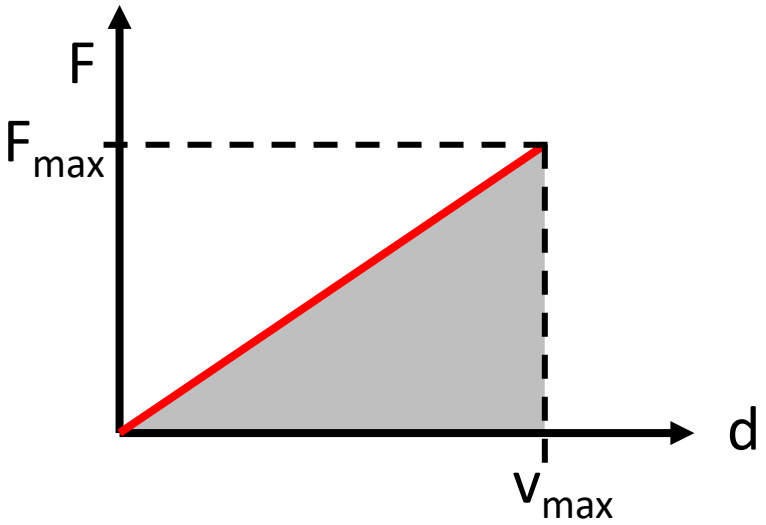


Energía potencial

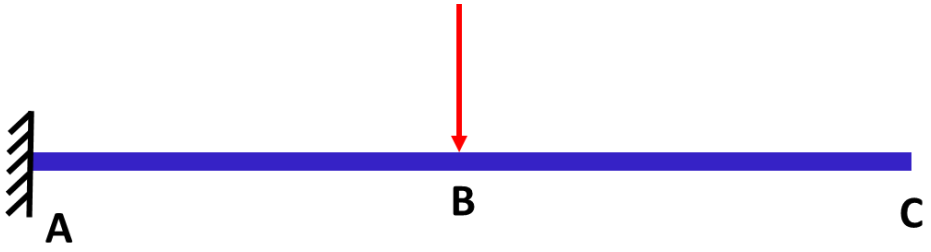


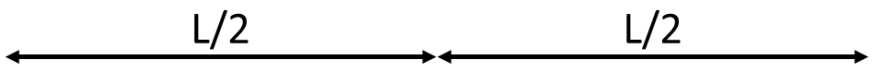


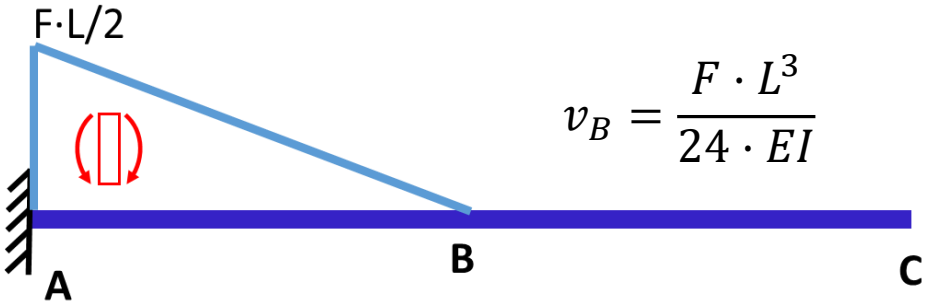
$$v_B = \frac{F \cdot L^3}{24 \cdot EI}$$



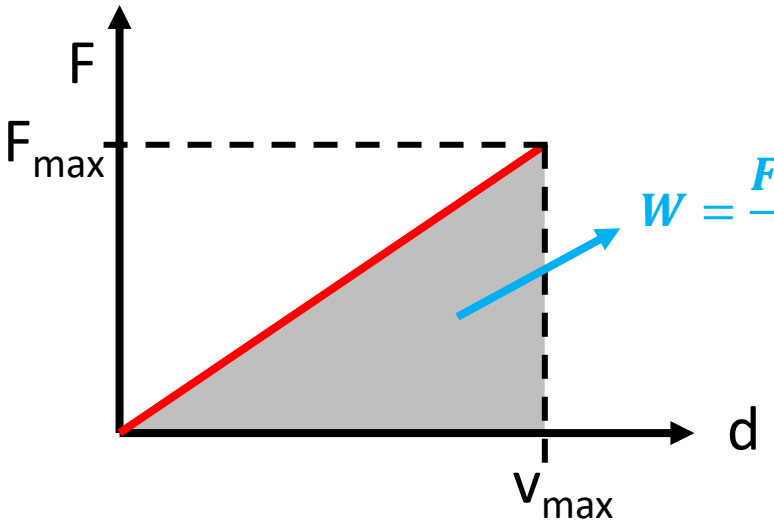
Energía potencial





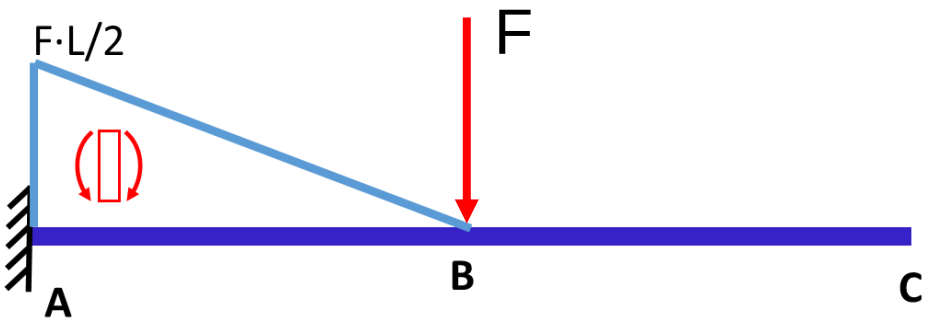


$$v_B = \frac{F \cdot L^3}{24 \cdot EI}$$



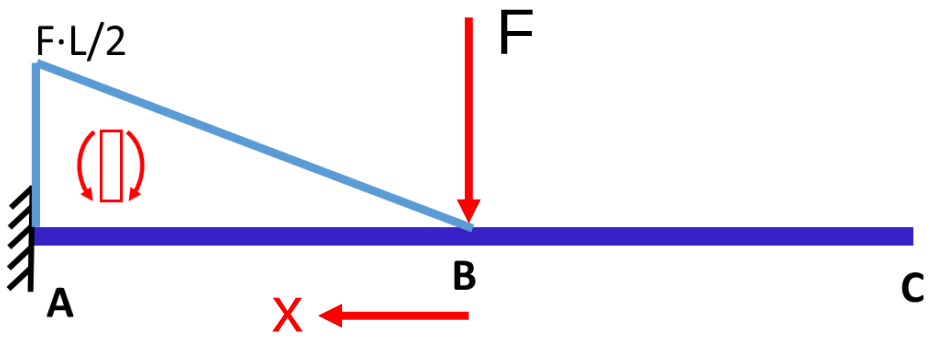
$$\boldsymbol{W = \frac{F \cdot v_c}{2} = \frac{F^2 \cdot L^3}{48 \cdot EI}}$$

Energía potencial



$$U = \int_A^C \left(\frac{M(x)}{2} \cdot \frac{M(x)}{EI}\right) dx = \int_A^C \frac{M^2(x)}{2 \cdot EI} dx$$

Energía potencial

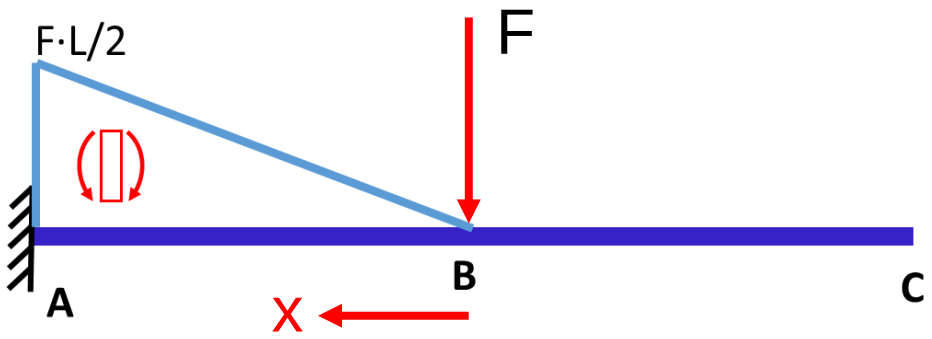


$$U = \int_A^C \left(\frac{M(x)}{2} \cdot \frac{M(x)}{EI}\right) dx = \int_A^C \frac{M^2(x)}{2 \cdot EI}\, dx$$

$$M(x) = F \cdot x$$

$$U = \int_0^{L/2} \frac{(F \cdot x)^2}{2 \cdot EI}\, dx = \frac{F^2 \cdot L^3}{48 \cdot EI}$$

Energía potencial



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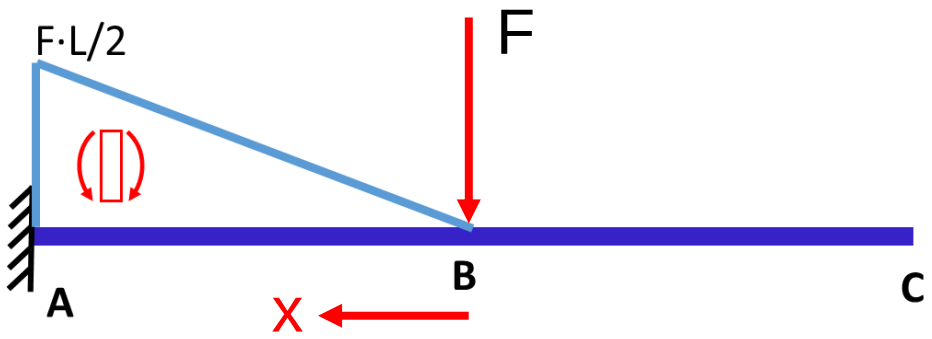
$$M(x) = F \cdot x$$

$$U = \int_0^{L/2} \frac{(F \cdot x)^2}{2 \cdot EI}\, dx = \frac{F^2 \cdot L^3}{48 \cdot EI}$$

$$v_B = \frac{\delta U}{\delta F} = \frac{F \cdot L^3}{24 \cdot EI}$$

Energía potencial



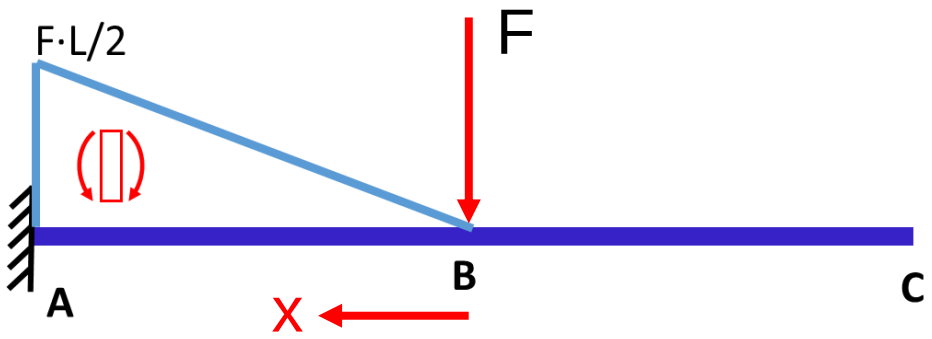
$$U = \int_A^C \left(\frac{M(x)}{2} \cdot \frac{M(x)}{EI}\right) dx = \int_A^C \frac{M^2(x)}{2 \cdot EI}\, dx$$

$$M(x) = F \cdot x$$

$$U = \int_0^{L/2} \frac{(F \cdot x)^2}{2 \cdot EI}\, dx$$

$$v_B = \frac{\delta U}{\delta F}$$

Energía potencial



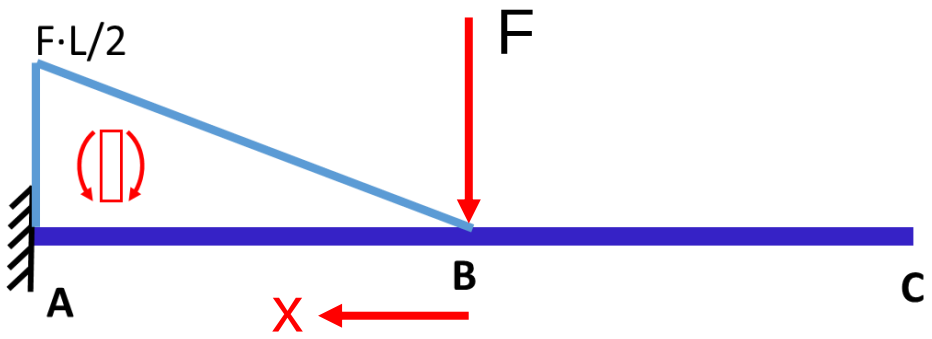
$$U = \int_A^C \left(\frac{M(x)}{2} \cdot \frac{M(x)}{EI}\right) dx = \int_A^C \frac{M^2(x)}{2 \cdot EI} dx$$

$$M(x) = F \cdot x$$

$$U = \int_0^{L/2} \frac{(F \cdot x)^2}{2 \cdot EI} dx$$

$$v_B = \frac{\delta U}{\delta F} = \int_0^{L/2} \frac{(F \cdot x) \cdot x}{EI} dx = \frac{F}{EI} \int_0^{L/2} x^2 dx$$

Energía potencial



$$U = \int_A^C \left(\frac{M(x)}{2} \cdot \frac{M(x)}{EI}\right) dx = \int_A^C \frac{M^2(x)}{2 \cdot EI}\, dx$$

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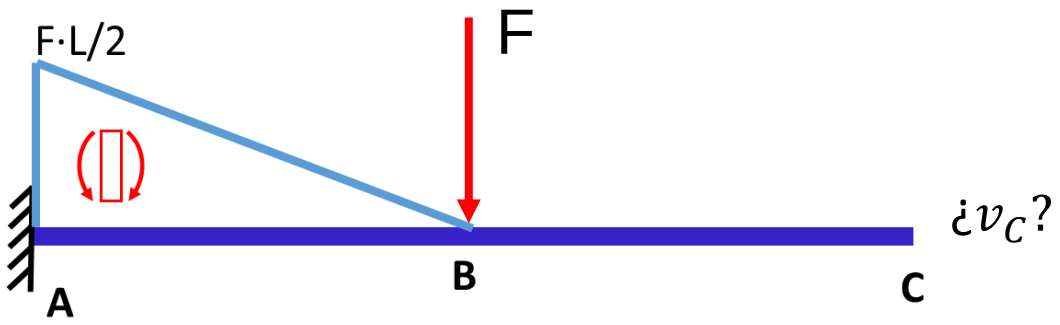
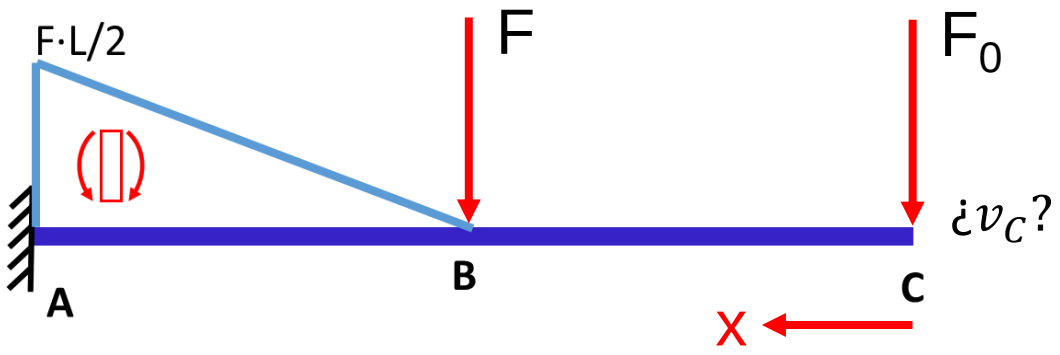
$$v_B = \frac{F \cdot L^3}{24 \cdot EI}$$

Energía potencial

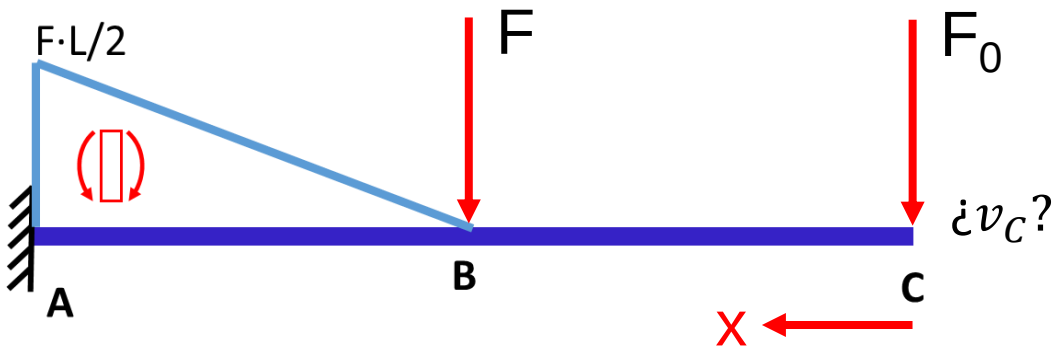


Energía potencial



$$U = \int_0^{L/2} \frac{(F_0 \cdot x)^2}{2 \cdot EI} dx + \int_{L/2}^{L} \frac{\left(F_0 \cdot x + F\left(x - \frac{L}{2}\right)\right)^2}{2 \cdot EI} dx$$

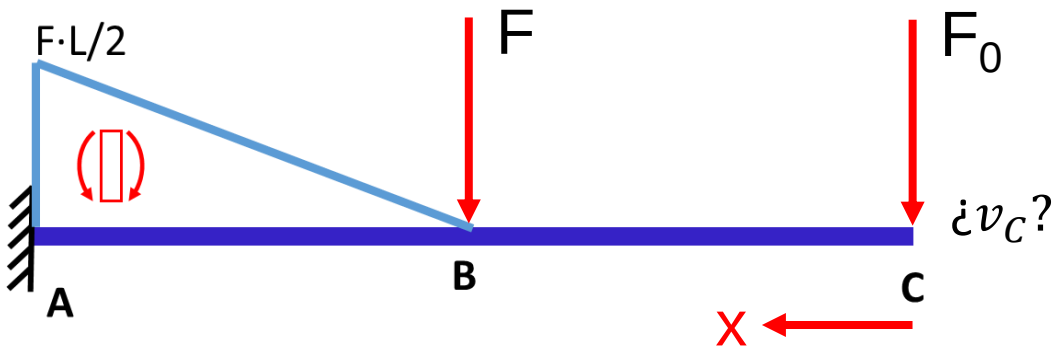
Energía potencial



$$U = \int_0^{L/2} \frac{(F_0 \cdot x)^2}{2 \cdot EI}\, dx + \int_{L/2}^{L} \frac{\left(F_0 \cdot x + F\left(x - \frac{L}{2}\right)\right)^2}{2 \cdot EI}\, dx$$

$$v_C = \frac{\delta U}{\delta F_0} = \int_0^{L/2} \frac{(F_0 \cdot x^2)}{EI}\, dx + \int_{L/2}^{L} \frac{\left(F_0 \cdot x^2 + F \cdot \left(x^2 - \frac{L \cdot x}{2}\right)\right)}{EI}\, dx$$

Energía potencial

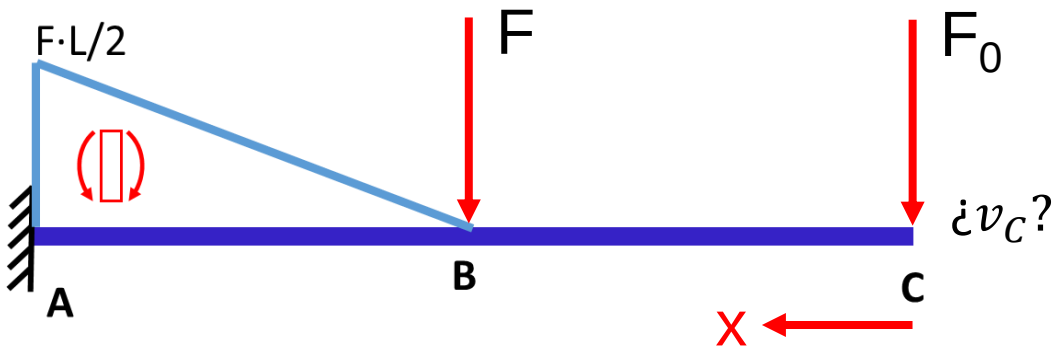


$$U = \int_0^{L/2} \frac{(F_0 \cdot x)^2}{2 \cdot EI}\, dx + \int_{L/2}^{L} \frac{\left(F_0 \cdot x + F\left(x - \frac{L}{2}\right)\right)^2}{2 \cdot EI}\, dx$$

$$v_C = \frac{\delta U}{\delta F_0} = \int_0^{L/2} \frac{(F_0 \cdot x^2)}{EI}\, dx + \int_{L/2}^{L} \frac{\left(F_0 \cdot x^2 + F \cdot \left(x^2 - \frac{L \cdot x}{2}\right)\right)}{EI}\, dx$$

$$v_C = \frac{F_0 \cdot L^3}{3 \cdot EI} + \frac{5 \cdot F \cdot L^3}{48 \cdot EI}$$

Energía potencial



$$U = \int_0^{L/2} \frac{(F_0 \cdot x)^2}{2 \cdot EI}\, dx + \int_{L/2}^{L} \frac{\left(F_0 \cdot x + F\left(x - \frac{L}{2}\right)\right)^2}{2 \cdot EI}\, dx$$

$$v_C = \frac{\delta U}{\delta F_0} = \int_0^{L/2} \frac{(F_0 \cdot x^2)}{EI}\, dx + \int_{L/2}^{L} \frac{\left(F_0 \cdot x^2 + F \cdot \left(x^2 - \frac{L \cdot x}{2}\right)\right)}{EI}\, dx$$

$$v_C = \frac{F_0 \cdot L^3}{3 \cdot EI} + \frac{5 \cdot F \cdot L^3}{48 \cdot EI} = \frac{5 \cdot F \cdot L^3}{48 \cdot EI}$$

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