

Control Engineering II

Topic 1: Z-Transform of Discrete-Time Systems

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2026

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Problem 1

Calculate the unilateral Z-transform of the unit ramp function:

$$x(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Take into account that the function can be rewritten considering the sampling period: $x(kT) = kT$, $k = 0, 1, 2, \dots$

Problem 2

Calculate the Z-transform of the sinusoidal function:

$$x(t) = \begin{cases} \sin \omega t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

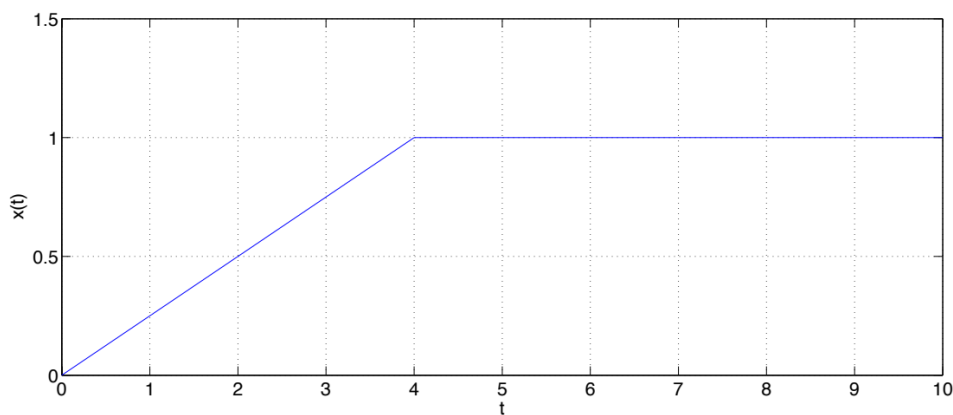
Problem 3

Calculate the Z-transform of the function:

$$f(k) = \begin{cases} a^{k-1}, & k = 1, 2, 3, \dots \\ 0, & k \leq 0 \end{cases}$$

Problem 4

Obtain the Z-transform of the curve shown in the figure assuming that the sampling time T is 1 second.



Problem 5

The function $y(k)$ is composed of a sum of functions $x(h)$, where $h = 1, 2, 3, \dots, k$, such that:

$$y(k) = \sum_{h=0}^k x(h) \quad k = 0, 1, 2, \dots$$

where $y(k) = 0$ for $k < 0$. Obtain the Z-transform of $y(k)$.

Problem 6

Given the Z-transforms of $\sin wt$ and $\cos wt$, obtain the Z-transform of $e^{-\alpha t} \sin wt$ and $e^{-\alpha t} \cos wt$ using the complex translation theorem.

Problem 7

Determine the final value $x(\infty)$ using the Final Value Theorem:

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-e^{-aT}z^{-1}}, \quad a > 0$$

Problem 8

Find $x(k)$ for $k = 1, 2, 3, 4, \dots$ when:

$$X(z) = \frac{10z + 5}{(z - 1)(z - 0.2)}$$

Problem 9

Obtain the inverse Z-transform for the following expression via partial fraction expansion:

$$X(z) = \frac{z^2 + z + 2}{(z - 1)(z^2 - z + 1)}$$

Problem 10

Obtain the inverse Z-transform for the following expression via partial fraction expansion:

$$X(z) = \frac{z + 2}{(z - 2)z^2}$$

Problem 11

Obtain the inverse Z-transform for the following expression via direct division and via partial fraction expansion:

$$X(z) = \frac{z(z + 2)}{(z - 1)^2}$$

Problem 12

Solve the following difference equation using the Z-transform method:

$$x(k + 2) + 3x(k + 1) + 2x(k) = 0, \quad x(0) = 0, \quad x(1) = 1$$

Problem 13

Solve the following difference equation using the Z-transform method leaving the result as a function of $x(0)$ and $x(1)$:

$$x(k+2) + (a+b)x(k+1) + abx(k) = 0$$

where a and b are constants and $k = 0, 1, 2, \dots$

Problem 14

Solve the following difference equation:

$$2x(k) - 2x(k-1) + x(k-2) = u(k)$$

$$u(k) = \begin{cases} 1, & k = 0, 1, 2, 3, \dots \\ 0, & k < 0 \end{cases}$$

where $x(k) = 0$ for $k < 0$.