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Control Engineering II

Topic 3:
Stability Analysis of Discrete-Time Systems

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Contents

- **Asymptotic Stability.** A system is said to be asymptotically stable if its response for any initial conditions tends to zero asymptotically in steady state. That is, the system response due to the initial conditions satisfies

$$\lim_{k\to\infty} y(k) = 0$$

- **Marginal Stability.** If the system response due to the initial conditions remains bounded but does not tend to zero, the system is said to be marginally stable.

- This second definition refers to the forced response of the system for a bounded input. An input is bounded when it satisfies

$$|u(k)| < b_u, \quad 0 < b_u < \infty, \quad k = 0, 1, 2, \ldots$$

- **BIBO Stability (Bounded-Input-Bounded-Output).**
  A system is said to be BIBO stable if its response to any bounded input remains bounded, that is, for any input satisfying the boundedness condition, the output satisfies:

$$|y(k)| < b_y, \quad 0 < b_y < \infty, \quad k = 0, 1, 2, \ldots$$

- It has been previously shown that the pole locations of the system transfer function determine the time response. This has strong implications for system stability.
- Consider the exponential sequence $\{p^k\}, k = 0, 1, 2, \ldots$ and its z-transform

$$\mathcal{Z}\{p^k\} = \frac{z}{z - p}$$

  where p is a real or complex number.
- The time sequence for large values of k is given by

$$|p^k| \rightarrow \begin{cases} 0 & |p| < 1 \\ 1 & |p| = 1 \\ \infty & |p| > 1 \end{cases} \tag{1}$$

Stability in the z-Plane

Pole Locations

- In general, any time sequence can be described by

$$f(k) = \sum_{i=1}^{n} A_i p_i^k, \quad k = 0, 1, 2, \ldots$$

whose Z-transform is given by

$$F(z) = \sum_{i=1}^{n} A_i \frac{z}{z - p_i}$$

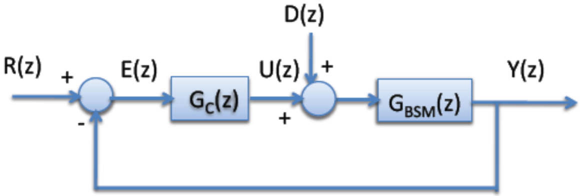
where A_i are the partial fraction coefficients and p_i are the poles in the z-domain.

- The sequence will be bounded if its poles are located inside or on the unit circle, and it decays exponentially if its poles are located inside the unit circle.

- In the absence of pole-zero cancellations, a linear time-invariant system is **asymptotically stable if the poles of its transfer function are located inside the unit circle**, and **marginally stable if the poles are located on the unit circle** and there are no repeated poles on the unit circle.

Internal Stability

- Stability as defined so far is applicable to open-loop and closed-loop systems with a single input and a single output. However, when the system has multiple inputs and outputs, this definition is not sufficient.
- For example, consider the following system with disturbances:



$$\frac{Y(z)}{R(z)} = \frac{G_C(z)G_{BSM}(z)}{1 + G_C(z)G_{BSM}(z)} \quad \frac{Y(z)}{D(z)} = \frac{G_{BSM}(z)}{1 + G_C(z)G_{BSM}(z)}$$

- **Definition:** A system is said to be **internally stable** when all transfer functions relating the system inputs and outputs are stable.

Determining System Stability

- The simplest method for determining the stability of a discrete-time system given by its transfer function is to find the system poles.
- One of the most commonly used numerical methods for this purpose is Newton's method.
- In the case of continuous-time systems, an alternative way to determine stability using a simple method is the **Routh-Hurwitz criterion**.
- For discrete-time systems, a similar method is the so-called **Jury test** or **Jury criterion** for systems whose transfer function is composed of polynomials with real coefficients (for polynomials with complex coefficients, the Schur-Cohn criterion can be used).

Jury Criterion

- Given a polynomial $F(z) = a_n z^n + a_{n-1} z^{n-1} + \ldots + a_1 z + a_0$, with $a_n > 0$,
- the roots of this polynomial lie inside the unit circle if and only if the following conditions are satisfied:
  - $F(1) > 0$
  - $(-1)^n F(-1) > 0$
  - $|a_0| < a_n$
  - $|b_0| > |b_{n-1}|$
  - $|c_0| > |c_{n-2}|$
  - $\ldots$
  - $|r_0| > |r_2|$

Jury Criterion

Cont.

- Where the terms $b_k, c_k, \ldots, r_k$ are calculated as follows:

$$b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix}, \quad k = 0, 1, \ldots, n-1$$

$$c_k = \begin{vmatrix} b_0 & b_{n-1-k} \\ b_{n-1} & b_k \end{vmatrix}, \quad k = 0, 1, \ldots, n-2$$

$$\ldots$$

$$r_0 = \begin{vmatrix} s_0 & s_3 \\ s_3 & s_0 \end{vmatrix}, \quad r_1 = \begin{vmatrix} s_0 & s_2 \\ s_3 & s_1 \end{vmatrix}, \quad r_2 = \begin{vmatrix} s_0 & s_1 \\ s_3 & s_2 \end{vmatrix}$$

- Based on these coefficients, the Jury table is constructed.

Jury Criterion
Table

Row	z^0	z^1	z^2	…	z^{n-k}	…	z^{n-1}	z^n
1	a_0	a_1	a_2	…	a_{n-k}	…	a_{n-1}	a_n
2	a_n	a_{n-1}	a_{n-2}	…	a_k	…	a_1	a_0
3	b_0	b_1	b_2	…	b_{n-k}	…	b_{n-1}	
4	b_{n-1}	b_{n-2}	b_{n-3}	…	b_k	…	b_0	
5	c_0	c_1	c_2	…	…	c_{n-2}		
6	c_{n-2}	c_{n-3}	c_{n-4}	…	…	c_0		
.	.	.	.	…	…			
.	.	.	.	…	…			
.	.	.	.	…	…			
$2n-5$	s_0	s_1	s_2	s_3				
$2n-4$	s_3	s_2	s_1	s_0				
$2n-3$	r_0	r_1	r_2					

$$F(z) = z^5 + 2.6z^4 - 0.56z^3 - 2.05z^2 + 0.0775z + 0.35$$

Row	z^0	z^1	z^2	z^3	z^4	z^5
1	0.35	0.0775	-2.05	-0.56	2.6	1
2	1	2.6	-0.56	-2.05	0.0775	0.35
3	b_o	b_1	b_2	b_3	b_4	
4	b_4	b_3	b_2	b_1	b_o	
5	c_o	c_1	c_2	c_3		
6	c_3	c_2	c_1	c_o		
7	d_o	d_1	d_2			

Row	z^0	z^1	z^2	z^3	z^4	z^5
1	0.35	0.0775	-2.05	-0.56	2.6	1
2	1	2.6	-0.56	-2.05	0.0775	0.35
3	-0.8775	-2.5728	-0.1575	1.854	0.8352	
4	b_4	b_3	b_2	b_1	b_o	
5	c_o	c_1	c_2	c_3		
6	c_3	c_2	c_1	c_o		
7	d_o	d_1	d_2			

Calculation of the third row

$$b_k = \begin{vmatrix} a_o & a_{n-k} \\ a_n & a_k \end{vmatrix}$$

$$b_o = \begin{vmatrix} a_o & a_5 \\ a_5 & a_o \end{vmatrix} = \begin{vmatrix} 0.35 & 1 \\ 1 & 0.35 \end{vmatrix} = -0.8775$$

$$b_1 = \begin{vmatrix} a_o & a_4 \\ a_5 & a_1 \end{vmatrix} = \begin{vmatrix} 0.35 & 2.6 \\ 1 & 0.0775 \end{vmatrix} = -2.5728$$

$$b_2 = \begin{vmatrix} a_o & a_3 \\ a_5 & a_2 \end{vmatrix} = \begin{vmatrix} 0.35 & -0.56 \\ 1 & -2.05 \end{vmatrix} = -0.1575$$

$$b_3 = \begin{vmatrix} a_o & a_2 \\ a_5 & a_3 \end{vmatrix} = \begin{vmatrix} 0.35 & -2.05 \\ 1 & -0.56 \end{vmatrix} = 1.854$$

$$b_4 = \begin{vmatrix} a_o & a_1 \\ a_5 & a_4 \end{vmatrix} = \begin{vmatrix} 0.35 & 0.0775 \\ 1 & 2.6 \end{vmatrix} = 0.8352$$

Jury Criterion

Example (Cont.)

The fourth row is identical to the third row, but in reverse order.

Row	z^0	z^1	z^2	z^3	z^4	z^5
1	0.35	0.0775	-2.05	-0.56	2.6	1
2	1	2.6	-0.56	-2.05	0.0775	0.35
3	-0.8775	-2.5728	-0.1575	1.854	0.8352	
4	0.8352	1.854	-0.1575	-2.5728	-0.8775	
5	c_o	c_1	c_2	c_3		
6	c_3	c_2	c_1	c_o		
7	d_o	d_1	d_2			

For row 5, the following expression is used again:

$$c_k = \begin{vmatrix} b_o & b_{n-k} \\ b_n & b_k \end{vmatrix}$$

Row	z^0	z^1	z^2	z^3	z^4	z^5
1	0.35	0.0775	-2.05	-0.56	2.6	1
2	1	2.6	-0.56	-2.05	0.0775	0.35
3	-0.8775	-2.5728	-0.1575	1.854	0.8352	
4	0.8352	1.854	-0.1575	-2.5728	-0.8775	
5	0.077	0.7143	0.2693	0.5151		
6	c_3	c_2	c_1	c_o		
7	d_o	d_1	d_2			

and so on...

1. The first row consists of the coefficients of $F(z)$ arranged in ascending order of the powers of z.
2. The number of rows in the table, $2n - 3$, is always odd, and the coefficients of each even row are the same as those of the odd row directly above it but in reverse order.
3. There are $n + 1$ conditions corresponding to the $n + 1$ coefficients of the table.
4. Conditions 3 to $2n - 3$ are computed using the coefficients of the first column of the Jury table together with the last coefficient of the preceding row. The intermediate coefficients of the last row are never used and therefore do not need to be calculated.

5. Conditions 1 and 2 are computed directly from $F(z)$. If either of the first two conditions is not satisfied, $F(z)$ has poles outside the unit circle and it is not necessary to construct the table.
6. Condition 3, with $a_n = 1$, requires the constant term of the polynomial to be less than unity in magnitude, since the constant term is the product of the roots and must therefore be less than 1.

- Determine the stability of the polynomial using the Jury criterion:

$$F(z) = z^5 + 2.6z^4 - 0.56z^3 - 2.05z^2 + 0.0775z + 0.35 = 0$$

- The Jury table is computed as follows:

Row	z^0	z^1	z^2	z^3	z^4	z^5
1	0.35	0.0775	–2.05	–0.56	2.6	1
2	1	2.6	–0.56	–2.05	0.0775	0.35
3	–0.8775	–2.5729	–0.1575	1.854	0.8325	
4	0.8325	1.854	–0.1575	–2.5729	–0.8775	
5	0.0770	0.7143	0.2693	0.5151		
6	0.5151	0.2693	0.7143	0.0770		
7	–0.2593	–0.0837	–0.3472			

- The first two conditions require evaluating $F(z)$ at $z = \pm 1$, and conditions 3 to 6 are obtained directly from the Jury table.
  1. $F(1) = 1 + 2.6 - 0.56 - 2.05 + 0.0775 + 0.35 = 1.4175 > 0.$
  2. $(-1)^5 F(-1) = (-1)(-1 + 2.6 + 0.56 - 2.05 - 0.0775 + 0.35) = -0.3825 < 0$
  3. $|0.35| < 1$
  4. $|-0.8775| > |0.8325|$
  5. $|0.0770| < |0.5151|$
  6. $|-0.2593| < |-0.3472|$
- Conditions 2, 5, and 6 do not satisfy the Jury criterion; therefore, there are poles outside the unit circle. This can be verified by factorization:

$$F(z) = (z - 0.7)(z - 0.5)(z + 0.5)(z + 0.8)(z + 2.5) = 0$$

- Determine the stable range of the gain K for a sampled-data system with a sampling period of 0.05 seconds, including a zero-order hold, and whose transfer function is

$$G(s) = \frac{10K}{s(s+10)}$$

- **Solution**.
  - The equivalent discrete-time transfer function of the hold-plant-sampler combination is:

$$\begin{aligned}
G_{BSM}(z) &= (1-z^{-1})\mathcal{Z}\{\mathcal{L}^{-1}[\frac{G(s)}{s}]\} \\
&= (1-z^{-1})\mathcal{Z}\{\mathcal{L}^{-1}[\frac{K}{s^2(s+10)}]\} \\
&= (1-z^{-1})\mathcal{Z}\{\mathcal{L}^{-1}[0.1K(\frac{10}{s^2} - \frac{1}{s} + \frac{1}{s+10}])\} \\
&= \frac{1.065 \times 10^{-2} K(z+0.847)}{(z-1)(z-0.606)} \qquad (2)
\end{aligned}$$

- If the system is connected with unity feedback, the characteristic equation $1 + G_{BSM}(z)$ is

$$z^2 + (1.065 \times 10^{-2}K - 1.606)z + 0.606 + 0.92 \times 10^{-3}K = 0$$

- The Jury stability conditions are:
  1. $F(1) = 1 + (1.0653 \times 10^{-2}K - 1.6065) + 0.6065 + 9.02 \times 10^{-3}K > 0 \equiv K > 0$
  2. $F(-1) = 1 - (1.0653 \times 10^{-2}K - 1.6065) + 0.6065 + 9.02 \times 10^{-3}K > 0 \equiv K < 1967.582$
  3. $|0.6065 + 0.0902K| < 1 \equiv +(0.6065 + 0.0902K) < 1$
     $-(0.6065 + 0.0902K) < 1$
     $\equiv -178.104 < K < 43.6199$
- From these three conditions it follows that

$$0 < K < 43.6199$$

Stability Analysis of a System Using MATLAB

- The Jury criterion is a method that is rarely used nowadays due to the availability of powerful numerical analysis tools.
- In MATLAB, let us see how stability analysis can be performed:
  1. Consider the following system $G(s) = \frac{2s+1}{s^2+4s+3}$
  2. Enter the model:

```
>> num=[2 1];
>> den=[1 4 3];
>> G=tf(num,den)
Transfer function:
2s+1
------------
s^2+4s+3
>> roots(den)
ans =
-3
-1
```

  3. The stability is then analyzed from the locations of the roots.

Root Locus in the z-Plane

- The characteristic equation of a closed-loop system with unity feedback is given by

$$1 + G(z)H(z) = 0$$

  where $H(z)$ is the feedback transfer function and $G(z)$ is the discrete-time transfer function of the system.
- The construction of the root locus is based on the same rules as in the continuous-time case, although its meaning and interpretation are different.

- Root locus construction rules:
    1. The number of root locus branches is equal to the number of poles of the open-loop transfer function $G(z)H(z)$.
    2. For positive values of K, points on the real axis belong to the root locus if the total number of poles and zeros located to the right of the point under consideration is an odd integer.
    3. The root locus starts ($K = 0$) at the poles and ends ($K = \pm\infty$) at the zeros of the open-loop transfer function or at $\pm\infty$.
    4. The angles of the root locus asymptotes that terminate at infinity are determined by
        - For $K > 0$ $\quad \gamma = \frac{(1+2k)\pi}{np_{GH} - nz_{GH}}$
        - For $K < 0$ $\quad \gamma = \frac{2k\pi}{np_{GH} - nz_{GH}}$
    5. The asymptotes intersect the real axis at

$$z_0 = \frac{\sum_{i=1}^{np} \mathrm{Re} p_i - \sum_{j=1}^{nz} \mathrm{Re} z_j}{np - nz}$$

6. The breakaway points of the root locus branches between two poles on the real axis (or the break-in points between two zeros on the real axis) can be determined by differentiating the loop gain K with respect to z. By setting this derivative equal to zero and solving the resulting equation, the roots obtained correspond to the branch breakaway or convergence points.
7. For $K > 0$, the departure angle of a branch from a complex pole is equal to 180 degrees minus the sum of the angles from the other poles plus the sum of the angles from the zeros (the angles may be positive or negative). For $K < 0$, it is 180 degrees plus the angle obtained for $K > 0$.
8. The root locus is symmetric with respect to the real axis.

9. The intersection of the root locus with the unit circle is determined using the Jury criterion applied to the closed-loop characteristic equation. The range of values of K that ensures stability is obtained.
10. Points on the root locus satisfy the magnitude criterion:

$$K = \frac{|z-p_1|\,|z-p_2|\,\ldots\,|z-p_{np}|}{|z-z_1|\,|z-z_2|\,\ldots\,|z-z_{nz}|}$$

11. Points on the root locus satisfy the angle criterion:

$$(1+2n)\pi = \sum_{i=1}^{np} \angle(z-p_i) - \sum_{j=1}^{nz} \angle(z-z_j), \quad K>0$$

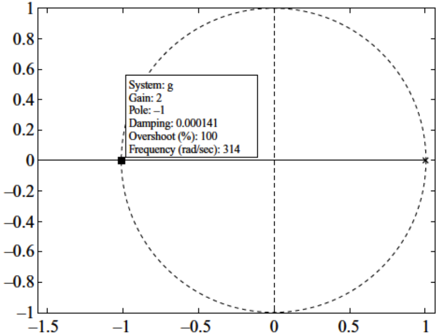
$$2n\pi = \sum_{i=1}^{np} \angle(z-p_i) - \sum_{j=1}^{nz} \angle(z-z_j), \quad K<0$$

Example 1

- Consider the first-order system $\frac{1}{z-1}$ with a proportional controller of gain K. The closed-loop characteristic equation is

$$1 + \frac{K}{z-1} = 0.$$

- The root locus has the form



Example 1
Cont.

- The system will be stable for gains that place the poles inside the unit circle.
- The critical gain corresponds to the point $(-1, 0)$ in

$$1 + \frac{K}{z-1} = 0$$

that is,

$$(z-1) + K = 0$$

and substituting the value of z at the critical point, $z = -1$, we obtain the critical gain

$$K_{cr} = 2.$$

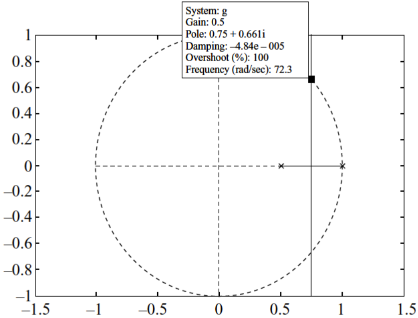
- Above this gain, the system becomes unstable.

Example 2

- Consider the second-order system $\frac{1}{(z-1)(z-0.5)}$ with a proportional controller of gain K. The closed-loop characteristic equation is

$$1 + \frac{K}{(z-1)(z-0.5)} = 0.$$

- The root locus takes the form



Example 2

Cont.

- The system will be stable for gains that place the poles inside the unit circle.
- The characteristic equation

$$1 + \frac{K}{(z-1)(z-0.5)} = 0$$

can be written as

$$(z-1)(z-0.5) + K = z^2 - 1.5z + K + 0.5 = 0$$

and since on the unit circle the two poles have magnitude 1 and are located on the vertical line between the poles $(z-1)$ and $(z-0.5)$, we obtain

$$z_{1,2} = 0.75 \pm j0.661$$

and from this the critical gain is $K_{cr} = 0.5$, which corresponds to these poles.

Root Locus Using MATLAB

- The command **rlocus()** is used:

  ```
  >> rlocus( tf([1],[1 -1.5 0.5 0],0.01))
  ```

  where 0.01 is the sampling period.

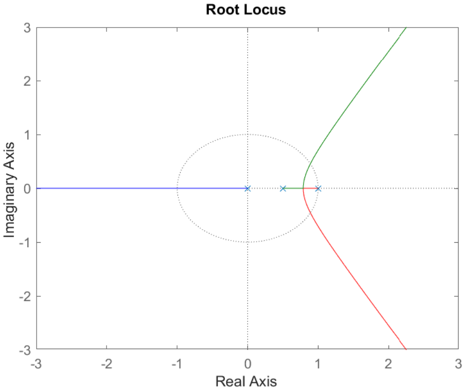


Figure 1: Root locus of the system.

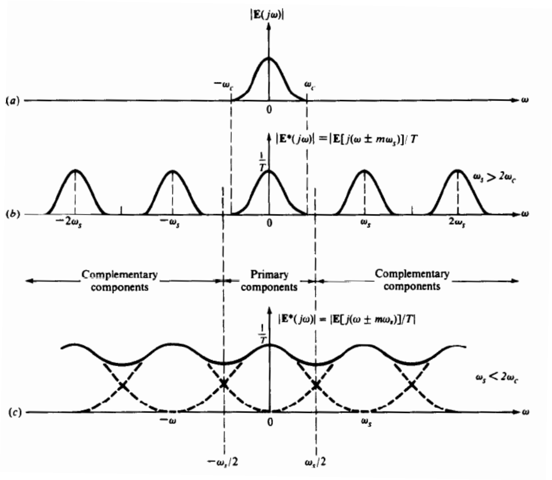


Figure 2: Fourier amplitude spectra.

- As discussed when introducing the sampling theorem, the effect of sampling is illustrated in the figure on the previous slide. It shows:
    1. The frequency spectrum of the continuous-time signal $e(t)$.
    2. The spectrum of the impulse-sampled signal $e^*(t)$ when sampled at a sampling frequency $\omega_s > 2\omega_c$, where ω_s is the sampling frequency and ω_c is the highest frequency component of the Fourier transform $E(j\omega)$.
    3. The spectrum of the impulse-sampled signal $e^*(t)$ when sampled at a sampling frequency $\omega_s < 2\omega_c$. In this case, the phenomenon of **aliasing or spectral overlap** between the fundamental and replicated spectral components can be observed.

Relationship Between the s-Plane and the z-Plane

Frequency Bands

- The frequency spectrum of the sampled signal $E^*(j\omega)$ with $\omega_s > 2\omega_c$ can be divided into a primary band and complementary bands that repeat periodically with period ω_s.
- This can be easily observed because, if we substitute s in $E^*(s)$ by $s + jl\omega_s$, with l an integer, we obtain

$$E^*(s + jl\omega_s) = \sum_{k=0}^{\infty} e(kT)e^{-kT(s+jl\omega_s)} = \sum_{k=0}^{\infty} e(kT)e^{-kTs}e^{-jkl\omega_s T}$$

and since $\omega_s = \frac{2\pi}{T}$, then $e^{-jkl\omega_s T} = e^{-j2\pi kl} = 1$, and therefore

$$E^*(s + jl\omega_s) = \sum_{k=0}^{\infty} e(kT)e^{-kTs} = E^*(s)$$

- A very important consequence is that if $E(s)$ has a pole (zero) at $s = s_1$, then $E^*(s)$ will have poles (zeros) at $s = s_1 \pm jl\omega_s$, where $l = 0, \pm 1, \pm 2, \ldots$.

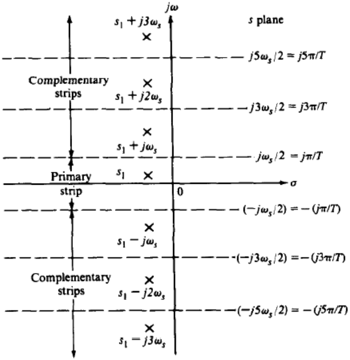


Figure 3: Effect of the bands on the poles.

- Since, in general, the physical system and the zero-order hold behave similarly to a low-pass filter, the effects of the high-frequency complementary bands are attenuated. Therefore, only the poles in the primary band need to be considered when designing the control system.
- Note that the primary pole s_1 and its complementary poles are mapped onto the same point in the z-plane (aliasing effect).

z-Plane

- Recall that the Laplace transform of a sampled-time signal $e^*(t)$ is given by

$$E^*(s) = \mathcal{L}\left[e^*(t)\right] = \sum_{k=0}^{\infty} e(kT)e^{-ksT}$$

where $e^*(t) = 0$ for $t < 0$ (unilateral transform) and $k = 0, 1, 2, \ldots$

- From this expression, the z-transform of the sampled signal is obtained directly by making the change of variable $z = e^{sT}$ (or $s = \frac{1}{T} \ln z$ to convert from the z-transform to the Laplace transform).
- Therefore,

$$E(z) = E^*(s)|_{z=e^{sT}} = \sum_{k=0}^{\infty} e(kT)z^{-k}$$

which is precisely the z-transform of the sampled signal $e^*(t)$.

Relationship Between the s-Plane and the z-Plane

- We have previously seen that the relationship between the sampled continuous-time transfer function in the s-domain and the discrete-time transfer function in the z-domain is given by

$$z = e^{sT}$$

- We now want to determine how this relationship maps points from the s-plane into the z-plane.

- Vertical lines in the s-plane are points for which $s = \sigma + j\omega$ has $\sigma =$ constant. These points correspond in the z-plane to those satisfying

$$|z| = e^{\sigma T} = \text{constant}$$

which form a circle centered at the origin.

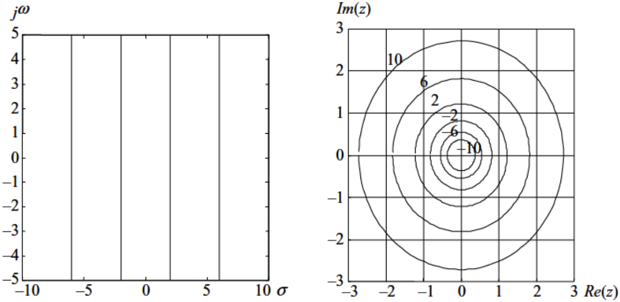


Figure 4: Vertical lines in the s-plane.

- Horizontal lines in the s-plane are points for which $s = \sigma + j\omega$ has $\omega =$ constant. These points correspond in the z-plane to those satisfying

$$\angle z = \omega T = \text{constant}$$

which form a radial line extending from the origin.

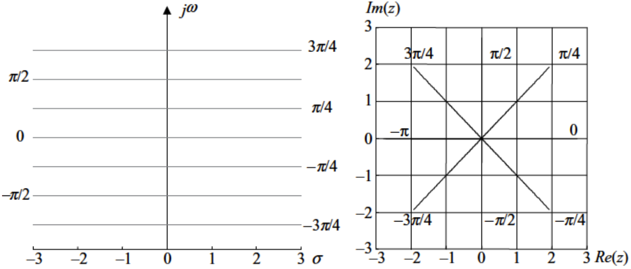


Figure 5: Horizontal lines in the s-plane.

Relationship Between the s-Plane and the z-Plane

Points with Equal Damping Ratio

- Given the relationship $z = e^{sT}$, for an underdamped second-order system with poles at

$$s_{1,2} = -\zeta\omega_n \pm j\omega_d,$$

the corresponding poles in the z-plane are

$$(z - e^{(-\zeta\omega_n + j\omega_d)T})(z - e^{(-\zeta\omega_n - j\omega_d)T}) = z^2 - \cos(\omega_d T)e^{-\zeta\omega_n T} z + e^{-2\zeta\omega_n T}$$

- As a result, the pole locations of the second-order system in the z-plane are

$$z_{1,2} = e^{-\zeta\omega_n T}\angle \pm \omega_d T = |z|\angle \pm \theta$$

or equivalently

$$\theta = \omega_d T = \omega_n \sqrt{1 - \zeta^2}$$

- Curves of constant ζ and constant ω_n in the z-plane are shown in the following figure:

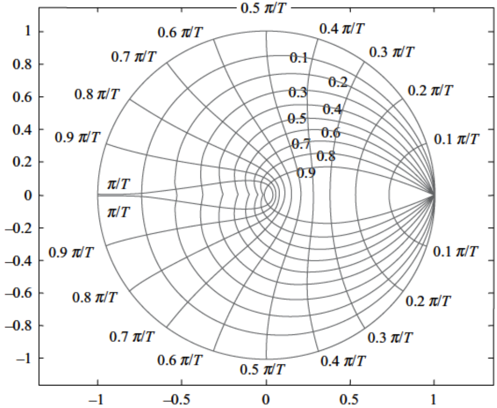


Figure 6: Constant ζ and ω_n contours in the z-plane.

- The shape of a system response depends on the locations of the poles of its characteristic equation and on the corresponding impulse responses associated with those poles.

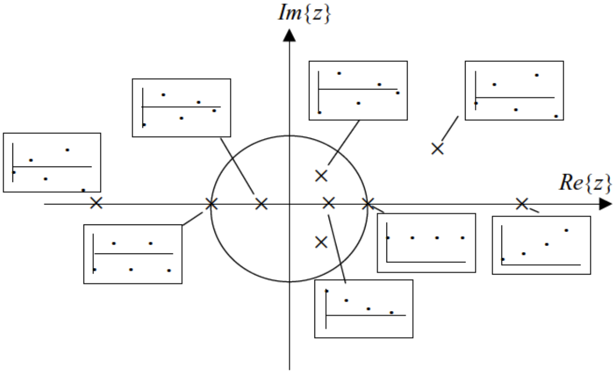


Figure 7: Pole locations and the corresponding impulse response of the system.

- The impulse response of a first-order system with an integrator (type 1)

$$G(z) = \frac{z}{z-1}$$

is

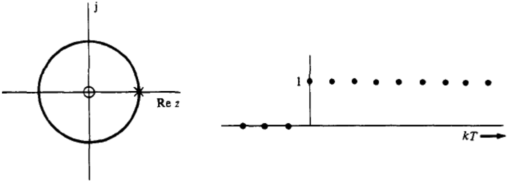


Figure 8: Pole locations and system response.

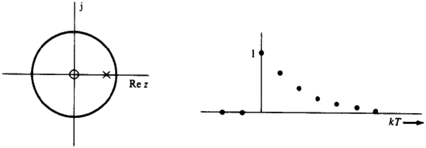
- This transfer function corresponds to a unit-step signal with transfer function

$$G(s) = \frac{1}{s}$$

- If the pole is moved inside the unit circle, the impulse response of the first-order system becomes that of a type-0 system,

$$G(z) = \frac{z}{z-d}, \qquad 0 < d < 1,$$

that is:



- This transfer function corresponds to a signal with transfer function

$$G(s) = \frac{1}{s+a}$$

where

$$d = e^{-aT}$$

Responses of Some Systems

Second-Order System

- If we discretize the second-order system

$$G(s) = \frac{s}{s^2 + a^2},$$

we obtain

$$G(z) = \frac{z(z - \cos(aT))}{z^2 + 2z\cos(aT) + 1},$$

whose impulse response is

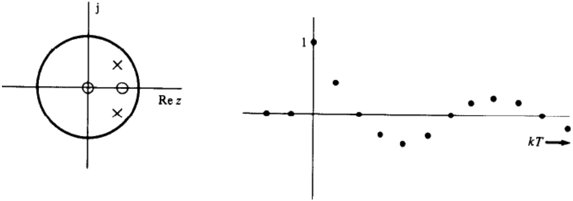


Figure 9: Pole locations and the impulse response of the system.

Impulse Response of a Discrete-Time System

First-Order System

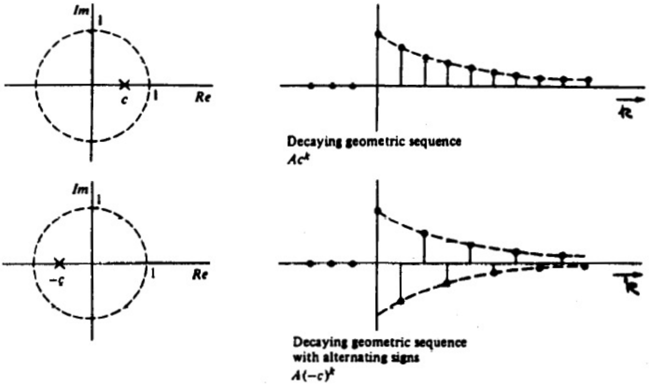


Figure 10: Pole locations and impulse responses for decaying geometric sequences.

Impulse Response of a Discrete-Time System

First-Order System

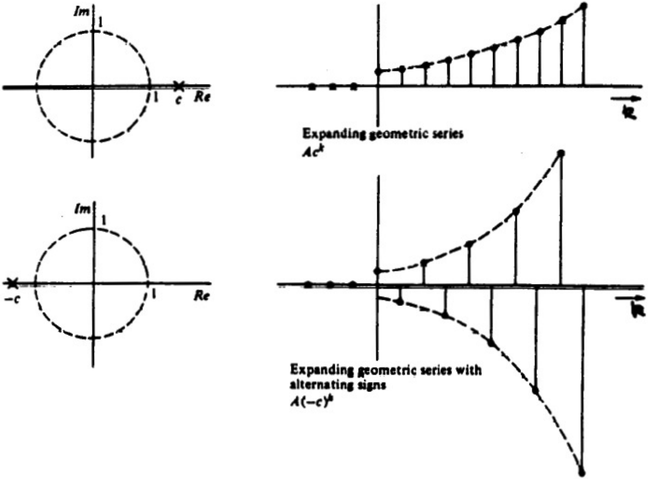


Figure 11: Pole locations and impulse responses for expanding geometric sequences.

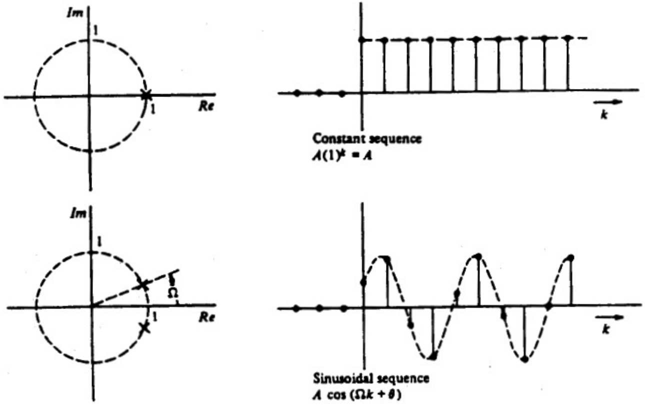


Figure 12: Pole locations and impulse responses for constant and sinusoidal sequences.

Impulse Response of a Discrete-Time System

Second-Order System

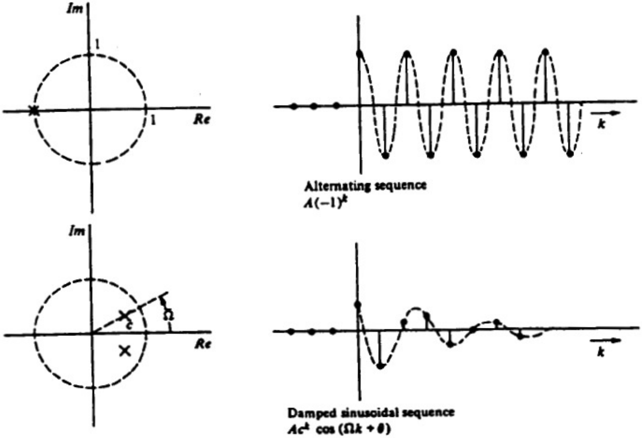


Figure 13: Pole locations and impulse responses for alternating and damped sinusoidal sequences.

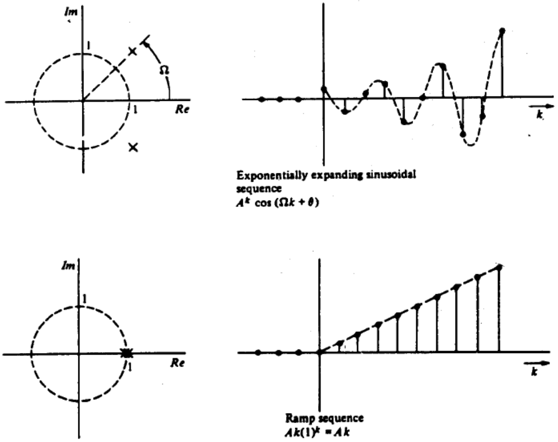


Figure 14: Pole locations and impulse responses for expanding sinusoidal and ramp sequences.