Control Engineering II

Topic 3: Stability Analysis of Discrete-Time Systems

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Problem 1

Examine the stability of the system given by the following characteristic equation:

$$F(z) = z^4 - 1.2z^3 + 0.07z^2 + 0.3z - 0.08 = 0$$

Solution:

We apply the Jury criterion and the following conditions must be satisfied:

$$F(z) = a_4 z^4 + a_3 z^3 + a_2 z^2 + a_1 z + a_0 = 0$$

1. $|a_0| < a_n$
2. $F(1) > 0$
3. $(-1)^n F(-1) > 0$
4. We construct the Jury stability table:

	z^0	z^1	z^2	z^3	z^4
1	a_0	a_1	a_2	a_3	a_4
2	a_4	a_3	a_2	a_1	a_0
3	b_0	b_1	b_2	b_3	
4	b_3	b_2	b_1	b_0	
5	c_0	c_1	c_2		

$$b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_4 & a_k \end{vmatrix} \qquad c_k = \begin{vmatrix} b_0 & b_{n-1-k} \\ b_{n-1} & b_k \end{vmatrix} \qquad n = 4$$

Problem 1

If we apply the criterion to the requested characteristic polynomial:

$$F(z) = z^4 - 1.2z^3 + 0.07z^2 + 0.3z - 0.08 = 0$$

1. $|a_0| < a_4, \;\; |-0.08| < 1$
2. $F(1) = 1 - 1.2 + 0.07 + 0.3 - 0.08 = 0.09 > 0$
3. $(-1)^4 F(-1) = 1 + 1.2 + 0.07 - 0.3 - 0.08 = 1.89 > 0$
4. We construct the Jury stability table:

	z^0	z^1	z^2	z^3	z^4
1	-0.08	0.3	0.07	-1.2	**1**
2	1	-1.2	0.07	0.3	**-0.08**
3	-0.994	1.176	-0.0756	-0.204	
4	-0.204	-0.0756	1.176	-0.994	
5	**0.946**	**-1.184**	**0.315**		

Applying the stability criteria:

$$|b_0| > |b_3| \;\; |-0.994| > |-0.204|$$

$$|c_0| > |c_2| \;\; |0.946| > |0.315|$$

Since all conditions are met, the given characteristic equation is stable. All roots are inside the unit circle.

Note: The characteristic polynomial can be rewritten in terms of its roots:

$$F(z) = (z - 0.8)(z + 0.5)(z - 0.5)(z - 0.4)$$

It can be observed that all roots are inside the unit circle of the z-plane.

Problem 2

Examine the stability of the system given by the following characteristic equation:

$$F(z) = z^3 - 1.1z^2 - 0.1z + 0.2 = 0$$

Solution

We apply the Jury criterion and the following conditions must be satisfied:
1. $a_n > |a_0|$: In this case: $a_3 > |a_0|\,,\;\; 1 > |0.2|$
2. $F(1) > 0$: $F(1) = 1 - 1.1 - 0.1 + 0.2 = 0$
This indicates that there is at least one root at $z = 1$. Therefore, the system is at best critically stable.
3. $(-1)^n F(-1) > 0$:
$(-1)^3 F(-1) = (-1)(-1 - 1.1 + 0.1 + 0.2) = (-1)(-1.8) = 1.8 > 0$
4. We construct the Jury stability table:

	z^0	z^1	z^2	z^3
1	a_0	a_1	a_2	a_3
2	a_3	a_2	a_1	a_0
3	b_0	b_1	b_2	

$$b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix} \qquad n = 3$$

	z^0	z^1	z^2	z^3
1	0.2	-0.1	-1.1	**1**
2	1	-1.1	-0.1	**0.2**
3	**-0.96**	b_1	**-0.12**	

Applying the stability criteria:

$$|b_0| > |b_2| \quad |-0.96| > |-0.12|$$

The fourth condition of the Jury Criterion is met.

It can be concluded that the system has a single root on the unit circle $(z = 1)$ and the other two are inside it. The system is critically stable.

Problem 3

Examine the stability of the system given by the following characteristic equation:

$$F(z) = z^3 - 1.3z^2 - 0.08z + 0.24 = 0$$

Solution

We apply the Jury criterion and the following conditions must be satisfied:
1. $a_n > |a_0|$: In this case: $a_3 > |a_0|\,, \;\; 1 > |0.24|$
2. $F(1) > 0$: $F(1) = 1 - 1.3 - 0.08 + 0.24 = -0.14 < 0$
The second condition is not met, so we conclude that the system is unstable.

Problem 4

Examine the stability of the system given by the following characteristic equation:

$$F(z) = z^3 - 1.3z^2 - 0.08z + 0.24 = 0$$

Solution

First, the feedback must be solved to obtain the closed-loop transfer function, which gives us the characteristic polynomial $\left(\frac{Y(z)}{X(z)} = \frac{G(z)}{1+G(z)H(z)}\right)$.

$$\frac{\frac{K(0.3679z+0.2642)}{(z-0.3679)(z-1)}}{1+\frac{K(0.3679z+0.2642)}{(z-0.3679)(z-1)}} = \frac{K(0.3679z+0.2642)}{(z-0.3679)(z-1)+K(0.3679z+0.2642)} =$$

$$= \frac{K(0.3679z+0.2642)}{z^2+(0.3679K-1.3679)z+(0.3679+0.2642K)}$$

The characteristic polynomial that defines the stability of the system is:

$$P(z) = z^2 + (0.3679K - 1.3679)z + (0.3679 + 0.2642K)$$

We apply the Jury criterion and the following conditions must be satisfied:

1. $a_n > |a_0|$: In this case: $a_2 > |a_0|$, $1 > |0.3679 + 0.2642K|$

$$0.3679 + 0.2642K < 1, \;\; 0.3679 + 0.2642K > -1$$

$$-5.1775 < K < 2.3925$$

2. $F(1) > 0$:
$F(1) = 1 + 0.3679K - 1.3679 + 0.3679 + 0.2642K = 0.6321K > 0$

$$K > 0$$

3. $(-1)^n F(-1) > 0$: $(-1)^2 F(-1) =$
$1 - 0.3679K + 1.3679 + 0.3679 + 0.2642K = -0.1037K + 2.7358 > 0$

$$0.1037K < 2.7358 \quad K < 26.3819$$

Since the polynomial is of second degree, the Jury Table has only one row. The values of K for which the system is stable are those for which all three conditions are satisfied: $0 < K < 2.3925$

Problem 5 (1/2)

Taking into account the system described by the following expression:

$$y(k) - 0.6y(k-1) - 0.81y(k-2) + 0.67y(k-3) - 0.12y(k-4) = x(k)$$

In this system $y(k)$ is the output and $x(k)$ is the input. Determine the stability of the system.

Solution

First we apply the Z-transform to obtain the equivalent transfer function:

$$Y(z) - 0.6z^{-1}Y(z) - 0.81z^{-2}Y(z) + 0.67z^{-3}Y(z) - 0.12z^{-4}Y(z) = X(z)$$
$$Y(z)(1 - 0.6z^{-1} - 0.81z^{-2} + 0.67z^{-3} - 0.12z^{-4}) = X(z)$$
$$\frac{Y(z)}{X(z)} = \frac{1}{(1 - 0.6z^{-1} - 0.81z^{-2} + 0.67z^{-3} - 0.12z^{-4})} = \frac{z^4}{z^4 - 0.6z^3 - 0.81z^2 + 0.67z - 0.12}$$

The characteristic polynomial of the system is as follows:

$$P(z) = z^4 - 0.6z^3 - 0.81z^2 + 0.67z - 0.12$$

We apply the Jury Criterion:
1. $a_n > |a_0|$: In this case: $a_4 > |a_0|$, $1 > |-0.12|$
2. $F(1) > 0$: $F(1) = 1 - 0.6 - 0.81 + 0.67 - 0.12 = 0.14 > 0$
3. $(-1)^n F(-1) > 0$: $(-1)^4 F(-1) = 1 + 0.6 - 0.81 - 0.67 - 0.12 = 0$
This indicates that there is at least one root at $z = -1$. Therefore, the system is at best critically stable.

4. We construct the Jury stability table:

	z^0	z^1	z^2	z^3	z^4
1	a_0	a_1	a_2	a_3	a_4
2	a_4	a_3	a_2	a_1	a_0
3	b_0	b_1	b_2	b_3	
4	b_3	b_2	b_1	b_0	
5	c_0	c_1	c_2		

Problem 5 (2/2)

Where:

$$b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_4 & a_k \end{vmatrix} \qquad c_k = \begin{vmatrix} b_0 & b_{n-1-k} \\ b_{n-1} & b_k \end{vmatrix} \qquad n = 4$$

	z^0	z^1	z^2	z^3	z^4
1	-0.12	0.67	-0.81	-0.6	**1**
2	1	-0.6	-0.81	0.67	**-0.12**
3	-0.9856	0.5156	0.9072	-0.5980	
4	-0.5980	0.9072	0.5196	-0.9856	
5	**0.6138**	c_1	**-0.5834**		

In this case, the table would be:

$$b_0 = \begin{vmatrix} a_0 & a_4 \\ a_4 & a_0 \end{vmatrix} = \begin{vmatrix} -0.12 & 1 \\ 1 & -0.12 \end{vmatrix} = -0.9856$$

$$b_3 = \begin{vmatrix} a_0 & a_1 \\ a_4 & a_3 \end{vmatrix} = \begin{vmatrix} -0.12 & 0.67 \\ 1 & -0.6 \end{vmatrix} = -0.5980$$

$$c_0 = \begin{vmatrix} b_0 & b_3 \\ b_3 & b_0 \end{vmatrix} = \begin{vmatrix} -0.9856 & -0.5980 \\ -0.5980 & -0.9856 \end{vmatrix} = 0.6138$$

$$c_2 = \begin{vmatrix} b_0 & b_1 \\ b_3 & b_2 \end{vmatrix} = \begin{vmatrix} -0.9856 & 0.5196 \\ -0.5980 & 0.9072 \end{vmatrix} = -0.5834$$

Applying the stability criteria:

$$|b_0| > |b_3| \quad |-0.9856| > |-0.5980|$$

$$|c_0| > |c_2| \quad |0.6138| > |-0.5834|$$

It can be concluded that the system has a single root on the unit circle ($z = -1$). The system is critically stable.

Problem 6 (1/2)

Obtain the root locus and the critical gain for the system whose transfer function is given by the following expression:

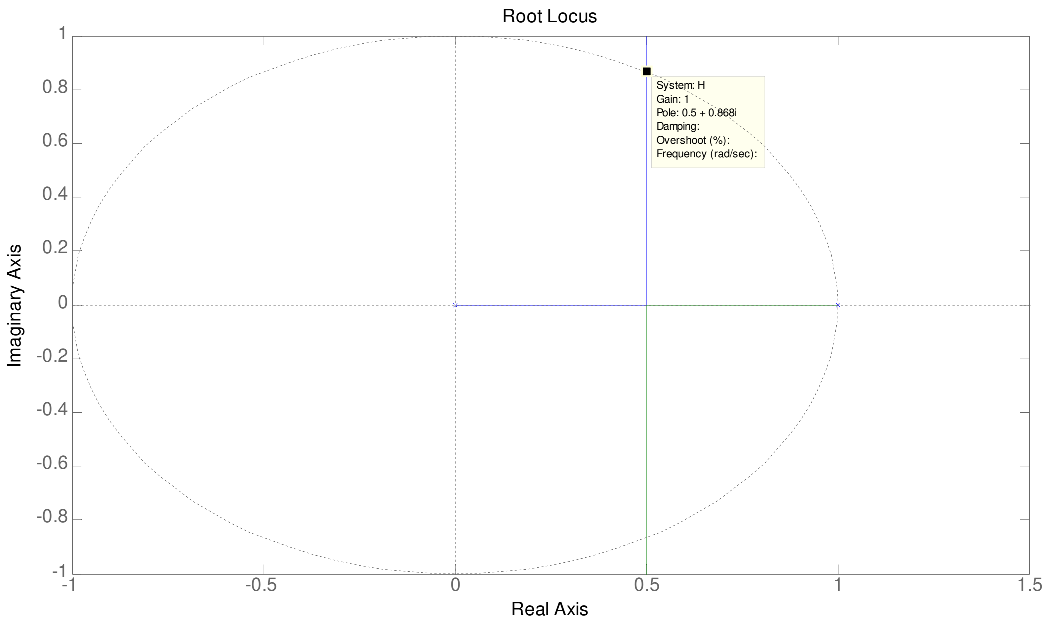
$$G(z) = \frac{1}{z(z-1)}$$

Solution

We draw the root locus applying the given rules. Only the rules with significant results are commented on.

1. Number of branches: 2
4. Asymptotes: $\gamma = (1+2k)\pi/(n_p - n_z) = \pi/2, 3\pi/2$
5. Centroid: $z_0 = (0+1)/2 = 0.5$

The root locus is represented in the following figure:



Problem 6 (2/2)

Now let's calculate the critical value of K, for which we write the characteristic equation:

$$(z-1)z + K = z^2 - z + K = 0$$

For any quadratic equation of the form: $ax^2 + bx + c = 0$ with roots x_1, x_2 it holds that $x_1 * x_2 = c/a$.
The squared modulus of two complex conjugate numbers located on the unit circle is equal to:

$$|z|^2 = z * \bar{z} = 1$$

On the unit circle, the poles are complex conjugates with a modulus equal to one. Since they are also roots of the second-degree equation, the following equation holds:

$$|z_{1,2}|^2 = K_{cr} = 1$$

where K_{cr} is the critical gain. The critical gain is equal to one and the conjugate poles are as follows:

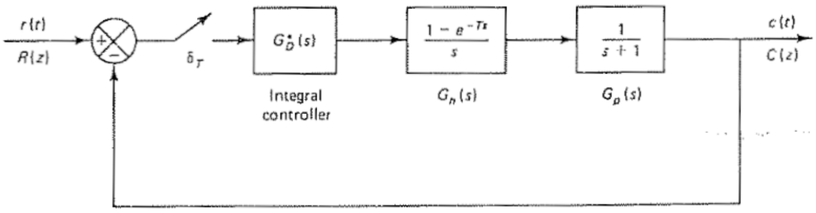
$$z_1 = 0.5 + j0.868 \quad z_2 = 0.5 - j0.868$$

Problem 7 (1/5)

Draw the root locus for the system with a digital controller in the figure, taking into account that the transfer function of the controller is given by the following expression:

$$G_D(z) = \frac{K}{1 - z^{-1}} = \frac{Kz}{z - 1}$$

Study the stability as a function of K for periods of 0.5 and 1 seconds. Calculate the poles of the system for K=2 in both cases.



Solution

First, the equivalent discrete function of the system must be calculated.

$$Z\left[G_b(s)G_p(s)\right] = Z\left[\frac{1 - e^{-Ts}}{s}\frac{1}{s+1}\right] = (1 - z^{-1})Z\left[\frac{1}{s(s+1)}\right] = ?$$

$$= (1 - z^{-1})Z\left[\frac{1}{s} - \frac{1}{s+1}\right] = \frac{z-1}{z}\left(\frac{z}{z-1} - \frac{z}{z - e^{-T}}\right) = \frac{1 - e^{-T}}{z - e^{-T}}$$

Taking into account the controller, the equivalent discrete transfer function is as follows:

$$G(z) = \frac{Kz}{z-1}\frac{1 - e^{-T}}{z - e^{-T}}$$

Since it is a system with unit feedback, the characteristic equation of the system is:

$$1 + G(z) = 1 + \frac{Kz}{z-1}\frac{1 - e^{-T}}{z - e^{-T}} = 0$$

Problem 7 (2/5)

Case 1: T=0.5sg

$$G(z) = \frac{0.3935Kz}{(z-1)(z-0.6065)}$$

We draw the root locus applying the given rules. Only the rules with significant results are commented on.
1. Number of branches: 2
4. Asymptotes: $\gamma = (1+2k)\pi/(n_p - n_z) = 180^o$
6. Breakaway points: First, express K as a function of z:

$$K = \frac{(z-1)(z-0.6065)}{0.3935z} = \frac{z^2 - 1.6065z + 0.6065}{0.3935z}$$

Differentiating and equating to zero yields the breakaway points:

$$\frac{dK}{dz} = \frac{dK}{dz}\left(\frac{z^2 - 1.6065z + 0.6065}{0.3935z}\right) = \frac{-z^2 - 0.6065}{0.3935z^2} = 0$$

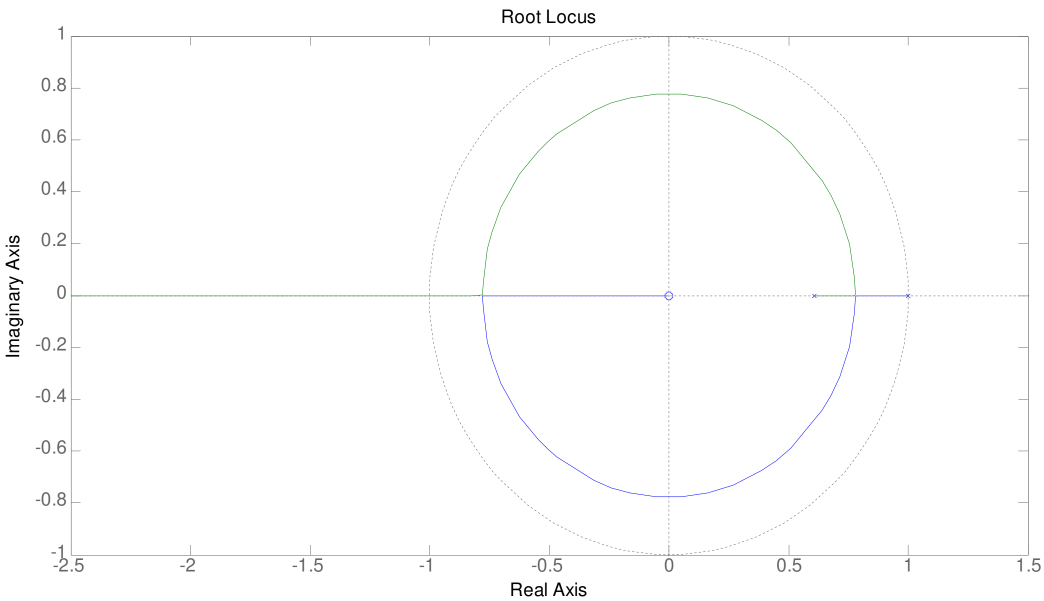
$$z = \pm 0.7788$$

Substituting z, we obtain the values of K:

$$z = 0.7788 \quad \rightarrow \quad K = 0.1244$$
$$z = -0.7788 \quad \rightarrow \quad K = 8.041$$

The first is the breakaway point and the second is the break-in point.

The root locus is represented in the following figure:



Problem 7 (3/5)

10. Now let's calculate the critical value of K, for which we apply the modulus criterion.

$$|K| = \left| \frac{|z-1|\,\left|z-e^{-T}\right|}{|z|\,\left|1-e^{-T}\right|} \right|$$

In this case:

$$|K| = \left| \frac{|z-1|\,|z-0.6065|}{|z|\,|0.3935|} \right|$$

The critical gain corresponds to the point $z=-1$. If we substitute in the previous equation:

$$|K| = \left| \frac{|-2|\,|-1.6065|}{|-1|\,|0.3935|} \right| = 8.165$$

For gains higher than this, the system will be unstable.
Finally, for K=2, the characteristic equation is:

$$z^2 + (0.3935K - 1.6065)z + 0.6065 = 0$$
$$z^2 - 0.8195z + 0.6065 = 0$$

Solving, the poles are located at:

$$z_1 = 0.4098 + j0.6623 \quad z_2 = 0.4098 - j0.6623$$

Case 2: T=1sg

In this case, the transfer function will be:

$$G(z) = \frac{0.6321Kz}{(z-1)(z-0.3679)}$$

We draw the root locus applying the given rules. Only the rules with significant results are commented on.

6. Breakaway points: First, express K as a function of z:

$$K = \frac{(z-1)(z-0.3679)}{0.6321z} = \frac{z^2 - 1.3679z + 0.3679}{0.6321z}$$

Differentiating and equating to zero yields the breakaway points:

$$\frac{dK}{dz} = \frac{dK}{dx}\left(\frac{z^2 - 1.3679z + 0.3679}{0.6321z}\right) = \frac{-z^2 - 0.3679}{0.6321z^2} = 0$$

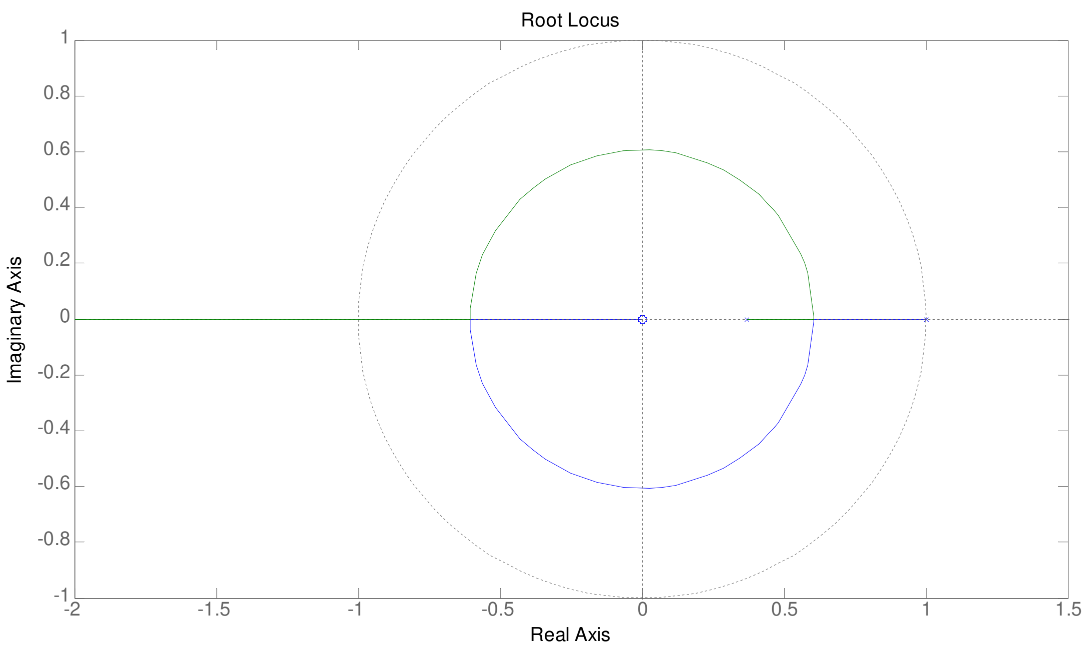
$$z = \pm 0.6065$$

Substituting z, we obtain the values of K:

$$z = 0.6065 \quad \rightarrow \quad K = 0.2449$$
$$z = -0.6065 \quad \rightarrow \quad K = 4.083$$

The first is the breakaway point and the second is the break-in point.

The root locus is represented in the following figure:



Problem 7 (5/5)

10. Now let's calculate the critical value of K, for which we apply the modulus criterion:

$$|K| = \left| \frac{|z-1|\,|z-0.3679|}{|z|\,|0.6321|} \right|$$

The critical gain corresponds to the point $z = -1$. If we substitute in the previous equation:

$$|K| = \left| \frac{|-2|\,|-1.3679|}{|-1|\,|0.6321|} \right| = 4.328$$

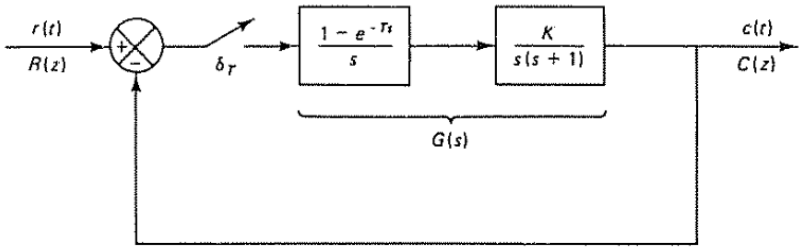
For gains higher than this, the system will be unstable.
Finally, for K=2 the poles will be, the characteristic equation is:

$$z^2 + (0.6321K - 1.3679)z + 0.3679 = 0$$
$$z^2 - 0.1037z + 0.3679 = 0$$

Solving, the poles are located at:

$$z_1 = 0.05185 + j0.6043 \quad z_2 = 0.05185 - j0.6043$$

Draw the root locus for the digital system in the figure.



Study the stability as a function of K for periods of 1, 2 and 4 seconds.

Solution

First, the equivalent discrete function of the system must be calculated (The discrete equivalent of this expression was already calculated in the first problem of Unit 2, and we take the result from there).

$$G(z) = Z\left[\frac{1-e^{-Ts}}{s}\frac{K}{s(s+1)}\right] = (1-z^{-1})Z\left[\frac{K}{s^2(s+1)}\right] =?$$
$$= \frac{K[(T-1+e^{-T})z^{-1} + (1-e^{-T}-Te^{-T})z^{-2}]}{(1-z^{-1})(1-e^{-T}z^{-1})}$$

Since it is a system with unit feedback, the characteristic equation of the system is:

$$1+G(z) = 1+\frac{K[(T-1+e^{-T})z^{-1} + (1-e^{-T}-Te^{-T})z^{-2}]}{(1-z^{-1})(1-e^{-T}z^{-1})} = 0$$

Case 1: T=1s

$$G(z) = \frac{0.3679K(z+0.7181)}{(z-1)(z-0.3679)}$$

We draw the root locus applying the given rules. Only the rules with significant results are commented on.

1. Number of branches: 2
4. Asymptotes: $\gamma = (1+2k)\pi/(n_p - n_z) = 180^{o}$
6. Breakaway points: First, express K as a function of z:

$$K = \frac{(z-1)(z-0.3679)}{0.3679(z+0.7181)} = \frac{z^2 - 1.3679z + 0.3679}{0.3679(z+0.7181)}$$

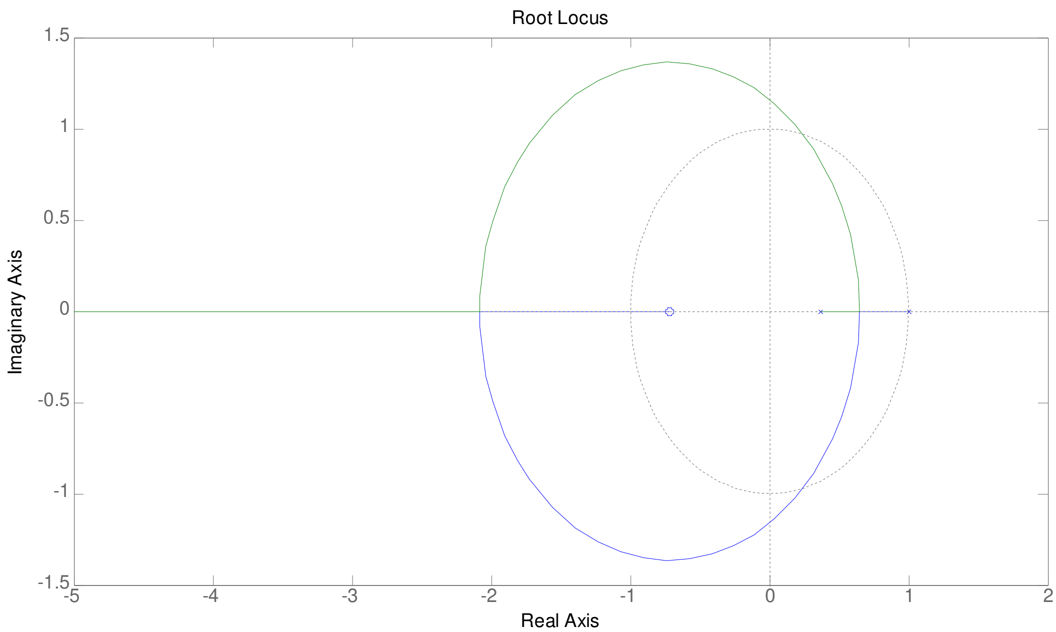
Differentiating and equating to zero yields the breakaway points:

$$\frac{dK}{dz} = \frac{dK}{dx}\left(\frac{z^2 - 1.3679z + 0.3679}{0.3679(z+0.7181)}\right) = 0$$

$$\frac{z^2 + 1.4362z - 1.3502}{0.3679(z+0.7181)^2} = 0$$

$$z_1 = 0.6479 \quad z_2 = -2.0841$$

The first is the breakaway point and the second is the break-in point. The root locus is represented in the following figure:



Now let's calculate the critical value of K, for which we calculate the characteristic equation:

$$1+G(z)=1+\frac{0.3679K(z+0.7181)}{(z-1)(z-0.3679)}=$$
$$=\frac{(z-1)(z-0.3679)+0.3679K(z+0.7181)}{(z-1)(z-0.3679)}=0$$

$$z^2+(0.3679K-1.3679)z+0.3679(1+0.7181K)=0$$

For any quadratic equation of the form: $ax^2+bx+c=0$ with roots x_1, x_2 it holds that $x_1 * x_2 = c/a$.
The squared modulus of two complex conjugate numbers is equal to:

$$|z|^2 = z * \bar{z} = 1$$

On the unit circle, the poles are complex conjugates with a modulus equal to one. Since they are also roots of the second-degree equation, the following equation holds:

$$|z_{1,2}|^2 = 0.3679(1+0.7181K_{cr}) = 1$$

The critical gain corresponds to:

$$K = 2.3925$$

For gains higher than this, the system will be unstable.

It can be calculated in another way: we know that the critical gain corresponds to two complex poles located on the unit circle, so their modulus will be equal to one. One way would be to give values to K until finding the desired poles, increasing or decreasing it based on the value of the modulus of the resulting poles.

$$K=2 \rightarrow \quad z^2-0.6321z+0.8963=0 \quad z_{1,2}=0.3161\pm j0.8924 \quad |z_{1,2}|=0.9467$$
$$K=3 \rightarrow \quad z^2-0.2642z+1.1605=0 \quad z_{1,2}=0.1321\pm j1.0691 \quad |z_{1,2}|=1.0773$$
$$K=2.3925 \rightarrow \quad z^2-0.4877z+1=0 \quad z_{1,2}=0.2438\pm j0.9698 \quad |z_{1,2}|=1$$

Case 2: T=2s

In this case, the transfer function will be:

$$G(z) = \frac{1.1353K(z+0.5232)}{(z-1)(z-0.1353)}$$

We draw the root locus applying the given rules. Only the rules with significant results are commented on.

6. Breakaway points: First, express K as a function of z:

$$K = \frac{(z-1)(z-0.1353)}{1.1353(z+0.5232)} = \frac{z^2-1.1353z+0.1353}{1.1353(z+0.5232)}$$

Differentiating and equating to zero yields the breakaway points:

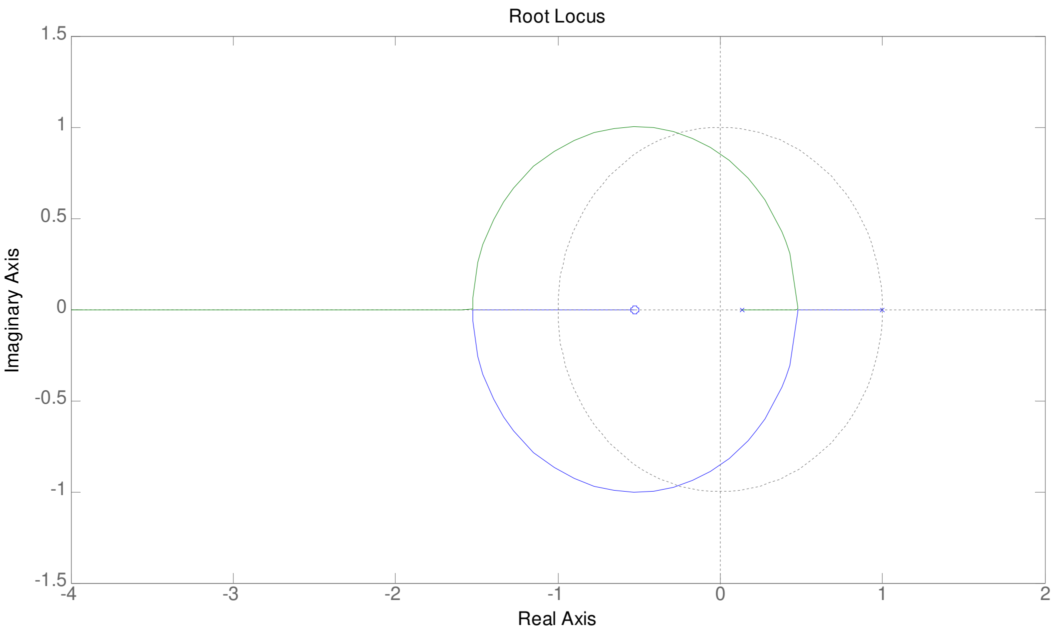
$$\frac{dK}{dz} = \frac{dK}{dx}\left(\frac{z^2-1.1353z+0.1353}{1.1353(z+0.5232)}\right) = 0$$

$$\frac{z^2+1.0464z-0.7287}{1.1353(z+0.5232)^2} = 0$$

$$z_1 = 0.4780 \quad z_2 = -1.5244$$

The first is the breakaway point and the second is the break-in point.

The root locus is represented in the following figure:



Problem 8 (5/7)

Now let's calculate the critical value of K, for which we calculate the characteristic equation:

$$1 + G(z) = 1 + \frac{1.1353K(z + 0.5232)}{(z-1)(z-0.1353)}$$

$$= \frac{(z-1)(z-0.1353) + 1.1353K(z+0.5232)}{(z-1)(z-0.1353)} = 0$$

$$z^2 + 1.1353(K-1)z + 1.1353 * 0.5232K + 0.1353 = 0$$

$$|z_{1,2}|^2 = 1.1353 * 0.5232K_{cr} + 0.1353 = 1$$

The critical gain corresponds to:

$$K = 1.4557$$

For gains higher than this, the system will be unstable.

Calculating it in the other way:

$$K = 2 \rightarrow \quad z^2 + 1.1353z + 1.3233 = 0 \quad z_{1,2} = -0.5676 \pm j1.0005 \quad |z_{1,2}| = 1.1503$$

$$K = 1 \rightarrow \quad z^2 + 0.7293 = 0 \quad z_{1,2} = \pm j0.8540 \quad |z_{1,2}| = 0.8540$$

$$K = 1.4557 \rightarrow \quad z^2 + 0.5174z + 1 = 0 \quad z_{1,2} = 0.2587 \pm j0.9660 \quad |z_{1,2}| = 1$$

Case 3: T=4 s

In this case, the transfer function will be:

$$G(z) = \frac{3.0183K(z+0.3010)}{(z-1)(z-0.0183)}$$

We draw the root locus applying the given rules. Only the rules with significant results are commented on.

6. Breakaway points: First, express K as a function of z:

$$K = \frac{(z-1)(z-0.0183)}{3.0183(z+0.3010)} = \frac{z^2-1.0183z+0.0183}{3.0183(z+0.3010)}$$

Differentiating and equating to zero yields the breakaway points:

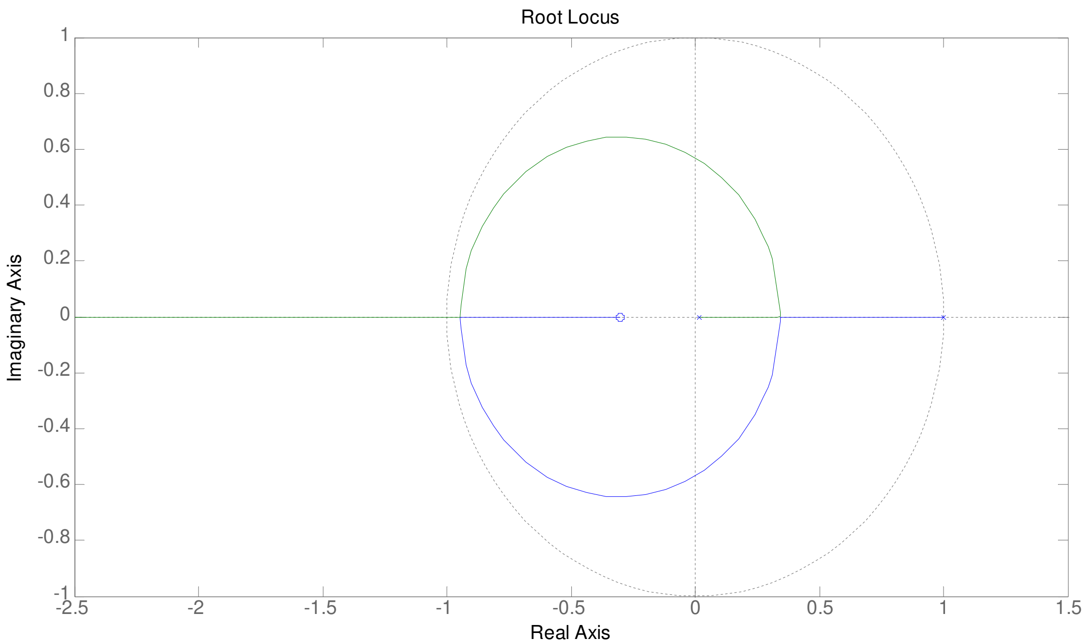
$$\frac{dK}{dz} = \frac{dK}{dx}\left(\frac{z^2-1.0183z+0.0183}{3.0183(z+0.3010)}\right) = 0$$

$$\frac{z^2+0.602z-0.3248}{3.0183(z+0.3010)^2} = 0$$

$$z_1 = 0.3435 \quad z_2 = -0.9455$$

The first is the breakaway point and the second is the break-in point.

The root locus is represented in the following figure:



Problem 8 (7/7)

Now let's calculate the critical value of K, for which we calculate the characteristic equation:

$$1 + G(z) = 1 + \frac{3.0183K(z+0.3010)}{(z-1)(z-0.0183)}$$
$$= \frac{(z-1)(z-0.0183) + 3.0183K(z+0.3010)}{(z-1)(z-0.0183)} = 0$$

$$z^2 + (3.0183K - 1.0183)z + 3.0183 * 0.3010K + 0.0183 = 0$$

In this case the modulus criterion is used, since they are not complex conjugate poles:

$$|K| = \left| \frac{|z-1|\,|z-0.0183|}{|z+0.3010|\,|3.0183|} \right|$$

The critical gain corresponds to the point $z = -1$. If we substitute in the previous equation:

$$|K| = \left| \frac{|-2|\,|-1.0183|}{|-1+0.3010|\,|3.0183|} \right| = 0.9653$$

The critical gain corresponds to:

$$K = 0.9653$$

For gains higher than this, the system will be unstable.
Substituting into the characteristic equation, the poles at critical gain are:

$$z_1 = -1 \quad z_2 = -0.8953$$

Note: In the case of having doubts about whether the poles at critical gain are or are not complex conjugates, one can always substitute the value of the critical gain obtained into the characteristic equation and check if the obtained results are valid.