Control Engineering II

Topic 7: Modeling and Analysis of Systems in State Space

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Problem 1
Linearize around an operating point the system given by the following state-space equations:

$$\begin{aligned}
\dot{x}_1 &= ax_1 + bx_2x_3 \\
\dot{x}_2 &= cx_3 + x_4u_1 \\
\dot{x}_3 &= u_1 + x_2x_4 \\
\dot{x}_4 &= x_1 + dx_3(1 - u_2)
\end{aligned}$$

$$\begin{aligned}
y_1 &= ex_1 + x_2 \\
y_2 &= x_3x_4
\end{aligned}$$

Problem 2
Calculate the three canonical forms: Controllable, Observable and Jordan, of the system whose transfer function is the following:

$$\frac{Y(z)}{U(z)} = \frac{z+1}{z^2 + 1.3z + 0.4}$$

Problem 3
Calculate the three canonical forms: Controllable, Observable and Jordan, of the system whose transfer function is the following:

$$\frac{Y(z)}{U(z)} = \frac{(z+0.5)(z-0.4)}{(z+1)(z-0.5)(z-0.5)}$$

Problem 4
Calculate the three canonical forms: Controllable, Observable and Jordan, of the system whose transfer function is the following:

$$\frac{Y(z)}{U(z)} = \frac{(z+0.3)(z-0.5)(z+0.8)}{(z-1)^2(z-0.5)(z+0.2)}$$

Problem 5

Obtain the matrices T that allow the conversion to the observable and controllable canonical forms given the following system:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$