Control Engineering II

Topic 10: State Observers

C.A. Monje, D. Copaci, M. Malfaz, J. Muñoz,
L. Mena, L. Moreno, S. Garrido

2026

uc3m | Universidad Carlos III de Madrid
OpenCourseWare

Problems

Problem 1
Suppose the system defined by the following equations:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} -0.8 & 1 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$[y_1(k)] = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Design a state observer for this system so that the error dynamics is as fast as possible (observer at the origin).

Problem 2
Suppose the system defined by the following equations:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1.2 \\ 1 & 0 & -0.8 \\ 0 & 1 & 0.3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 2.1 \\ -0.2 \\ 1.1 \end{bmatrix} u(k)$$

$$[y_1(k)] = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

Design a state observer for this system so that the observer poles are a double one at the origin and another at z=-0.1.

Problem 3
Suppose the system defined by the following equations:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.5 & 1 \\ -0.1 & 1.2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$[y_1(k)] = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Design a state observer for this system so that the error dynamics is as fast as possible (observer at the origin). First convert the system to the observable canonical form.

Problem 4
Suppose the system defined by the following equations:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.4 & 1.1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$
$$[y_1(k)] = \begin{bmatrix} 1.3 & 0.2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Design a state observer for this system so that the error dynamics is as fast as possible (observer at the origin). First convert the system to the observable canonical form.

Problem 5
Suppose the system defined by the following equations:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.8 & -0.5 \\ 1 & 0.5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.6 \\ 0.1 \end{bmatrix} u(k)$$
$$[y_1(k)] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Design a reduced-order state observer for this system so that the error dynamics is as fast as possible (observer at the origin). Assume for this that only variable 1 is observable.

Problem 6
Suppose the system defined by the following equations:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.2 \\ 0 & 0.5 & 0.1 \\ 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0.01 \\ 0.1 \\ 0.2 \end{bmatrix} u(k)$$
$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

Design a reduced-order state observer for this system so that the error dynamics is as fast as possible (observer at the origin). Assume for this that only variables 1 and 2 are observable.