Control Engineering II

Practice 2:
Controller Design by Direct Synthesis

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1. Digital control systems

Controllers in a control system can be implemented using several techniques and components, e.g.: mechanical systems, hydraulic systems, analog or digital electronic circuits, and also microprocessors. Compared with analog components, the latter have a relatively slow response; however, the continuous development of faster microprocessors means that, in practice, this limitation is becoming negligible, providing great versatility in the design and tuning of the controller parameters. In general, the typical topology to be considered in digital control systems is shown in Figure 1.

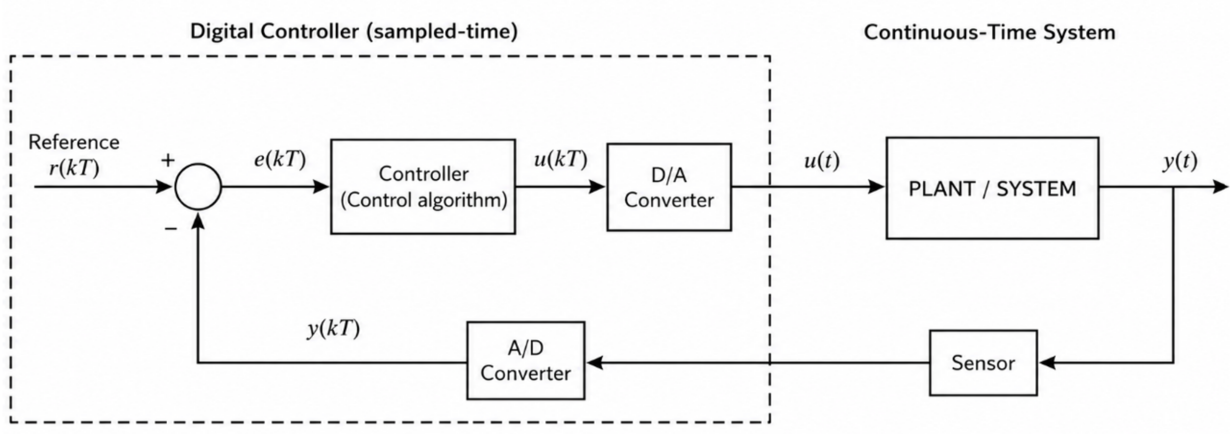

Figure 1: Digital control system with microprocessor

In general, digital control systems require the use of analog-to-digital (A/D) and digital-to-analog (D/A) converters, since the I/O signals of the computer are digital and the plant or process to be controlled usually uses analog signals (Figure 1). Both converters sample the signal every T_s seconds, where T_s is the sampling time and is one of the most important parameters to consider in the design of digital control systems.

In the digital control system shown in Figure 1, the controller is a high- or low-level program executed on a dedicated processor with the minimum hardware equipment required to perform the control tasks, or, as in our case, on a personal computer.

2. Design of digital control algorithms

2.1 Discretization Principles

2.1.1. Discretization using the Z-transform of G(s)

The effects of the discrete processing of signals in a digital control system, and therefore of the A/D and D/A converters, can be modeled by means of a zero-order sampler and hold, as shown in Figure 2.

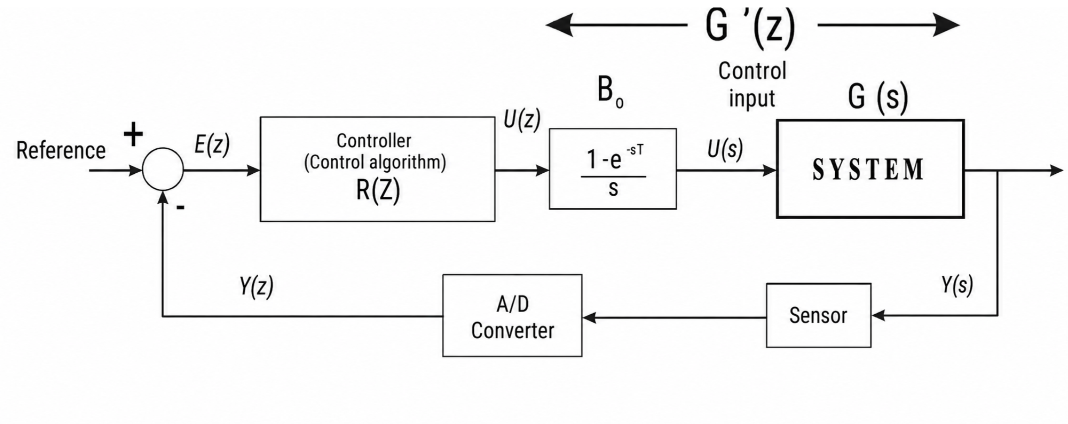

Figure 2: Modeling of a digital control system with a zero-order sampler and hold

In this case, the Z-domain transfer function of the process-hold system is given by:

$$G^{'}(z) = Z\left\{\frac{1-e^{-sT_s}}{s}G(s)\right\} = (1-z^{-1})Z\left\{\frac{G(s)}{s}\right\}. \quad (1)$$

2.1.2. Sampling time

As observed in the previous equations, in the design of digital controllers, the appropriate selection of the sampling period T_s is important and depends on the system. A value of T_s that is too large degrades the stability of the system, leading to loss of information about the signals if they change rapidly. On the other hand, if the value of T_s is too small, faster A/D converters with higher resolution will be required.
In order to properly reconstruct the sampled signal, Shannon's theorem establishes a criterion for selecting T_s, that is:

$$\omega_s > 2\omega_b \quad (2)$$

where ω_s is the sampling frequency and ω_b is the bandwidth of the signal, both measured in rad/s.

There are also practical rules for determining the sampling time. One commonly used by control engineers is to satisfy the following relationship:

$$T_s < \frac{T_{dom}}{10} \tag{3}$$

where T_s is the sampling period and T_{dom} is the dominant time constant of the system. In this practice, we choose $T_s = T_{dom}/10$. Industrial processes are usually modeled with varying degrees of accuracy as first-order systems. In these systems, the time constant corresponds to the time at which the output reaches 0.63 of its final value.
In general, sampling intervals of 1 second are suitable for flow, pressure, level, and temperature processes, whereas electromechanical systems require sampling times on the order of milliseconds.

3. Position control of a DC motor

When different actuation methods and position and velocity measurements are considered, motors provide practical examples of characteristic situations in real-time control systems; furthermore, they are relatively inexpensive and provide a good learning environment. On the other hand, the software developed for motor control can be applied to other control problems by making minor changes to its configuration. Figure 3 shows a simplified control system for a direct current motor.

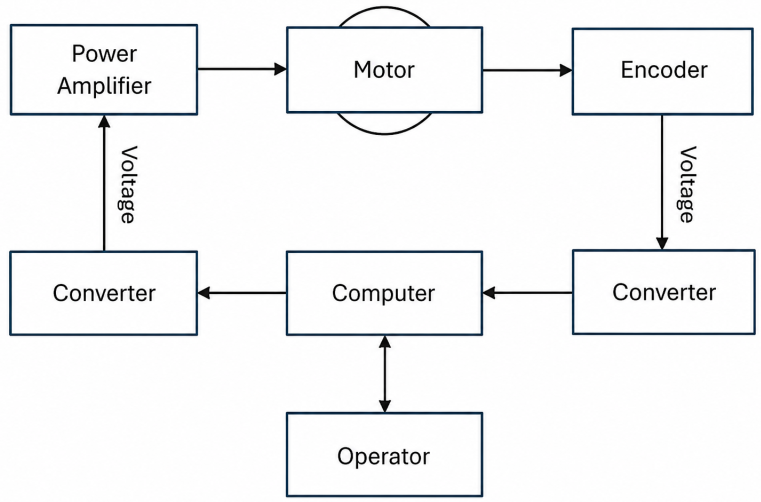

Figure 3: Digital control system of a DC motor

3.1. The physical system

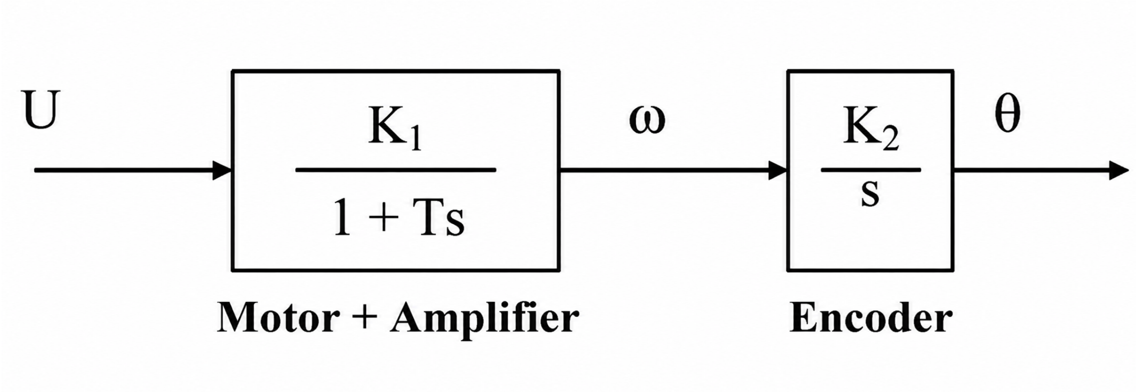

Figure 4: Motor system with position output

The open-loop transfer function of the motor with position output to be considered, as shown in Figure 4, is given by:

$$\frac{\theta(s)}{U(s)} = \frac{K_1}{(1+Ts)}\frac{K_2}{s} \tag{4}$$

where K_1, K_2 and T are the gains of the motor and the encoder, and the motor time constant, respectively. The identification of the system parameters is performed by analyzing the time response of the system in open loop.

4. Simulation work

Given the direct current motor with the following parameters, according to the scheme in Figure 4: $K_1K_2 = 4.20$ and $T = 0.12$. Design, using direct synthesis, the simplest controller that makes the closed-loop transfer function have unit gain and the following dominant poles:

- p_1 = 0.9
- p_2 = 0.8

To check the results obtained in this section, complete the file P2.mlx and run it using the "Run" button. To do this, first set the current Matlab directory to the folder of the practice session (see Figure 5) and then open the file P2.mlx from the current directory.

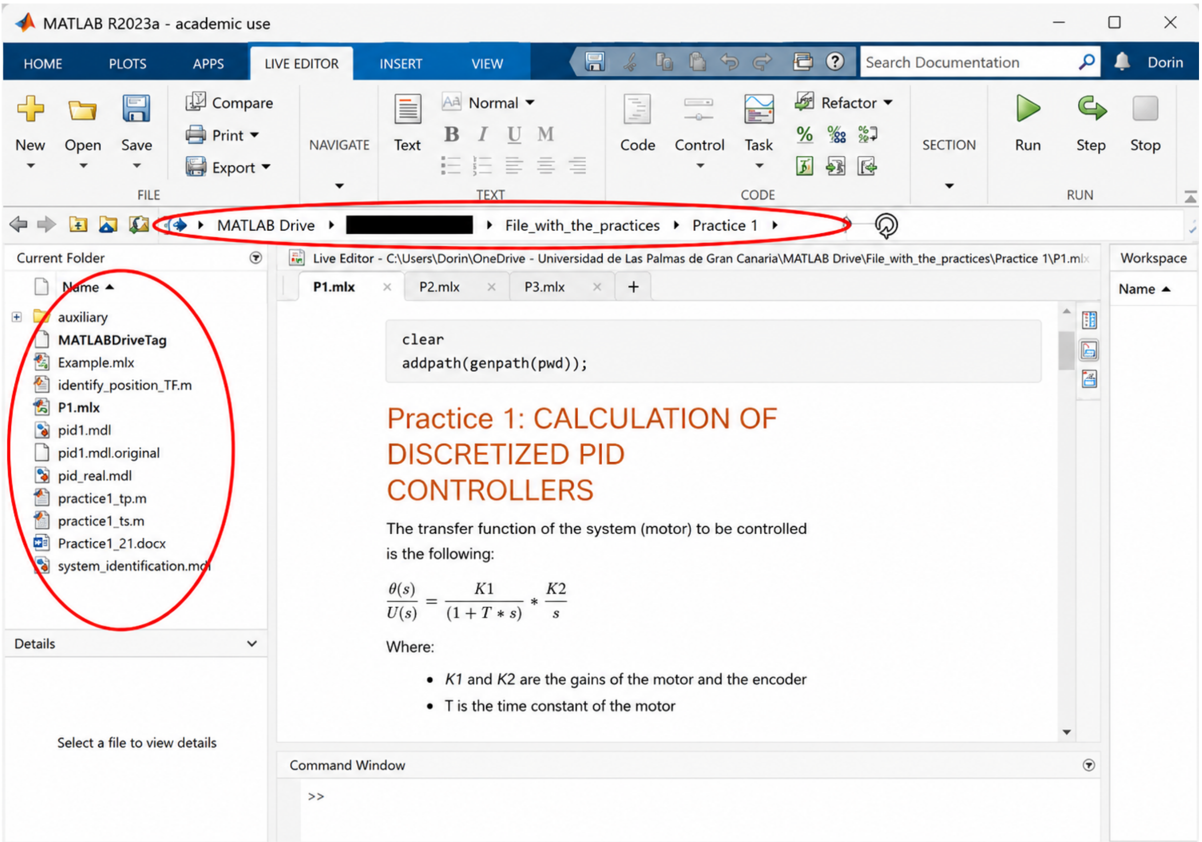

Figure 5: Matlab interface

1. Obtain the expression of the discrete-time controller using the direct synthesis method.
2. Launch the Matlab Simulink tool and open the file "*truxal.mdl*". Once this has been done, become familiar with each of its blocks, identifying the equivalence between the system in Figure 4 and its feedback scheme represented in Simulink.
3. Assign the values to each of the blocks according to the given motor parameters and those obtained for the controller.
4. Analyze the results obtained.
   - Plot both the output and the control action.
   - Measure the settling time.