Control Engineering II

Practice 3:
Position Control of a DC Motor Using State Feedback

C.A. Monje, D. Copaci, M. Malfaz, J. Muñoz,
L. Mena, L. Moreno, S. Garrido

2026

uc3m | Universidad Carlos III de Madrid
OpenCourseWare

1. Digital control systems

A DC motor can be approximately modeled as a second-order system, described by the following transfer function:

$$\frac{\theta(s)}{U(s)} = \frac{K_1}{(1+Ts)}\frac{K_2}{s} \tag{1}$$

where K_1, K_2 and T are the gains of the motor and the encoder, and the motor time constant, respectively. The identification of the system parameters is performed by analyzing the time response of the system in open loop.

If the open-loop time responses of velocity and position of the system are analyzed, it is possible to identify the parameters of the previous transfer function.

From equation (1), the state-variable equations of the motor are obtained:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Bu \tag{2}$$

$$y = C \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{3}$$

where: $x_1 = \theta$; $\dot{x}_2 = \theta$; $u = E_a$

$$A = \begin{bmatrix} 0 & K_2 \\ 0 & \frac{-1}{T} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{K_1}{T} \end{bmatrix} \tag{4}$$

And for the output we have:

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{5}$$

1.1 Sampling time

To perform digital control of the motor, it is necessary to obtain the discrete state equations of the system. For this purpose, an appropriate sampling time must be selected, depending on the system to be controlled. This value can be chosen by applying theoretical criteria such as Shannon's theorem or, as in this case, the practical criterion stating that the sampling time must be at least 10 times smaller than the system time constant.

2. Simulation work

2.1. Objectives of the practice

Design the position control system of a motor using:

- State feedback.
- State feedback with static gain adjustment.

2.2. Conversion to discrete equations.

Calculate the discrete state matrices G and H of the system:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} u(k) \tag{6}$$

For this purpose, use the Matlab command "c2d" with a sampling time (Ts) equal to $Ts = Tdom/10$. Example:

- $>> SYS = SS(A, B, C, D)$
- $>> SYSD = c2d(SYS, Ts)$
- $>> [G, H, C, D] = ssdata(SYSD)$

4. Simulation work

Given the direct current motor with the following parameters: $K_1 = 4$, $K_2 = 1$ and $T = 0.2$. Obtain the state feedback matrix K required for the closed-loop system to satisfy the following specifications:

- M_p = 15 %
- n_s = 12 samples

In order to solve the exercise, the system must first be discretized following the steps explained in Section 2.2.
Note: It is possible to use the Matlab commands "place" or "acker" to calculate the value of the elements of the feedback matrix. Example:

- $>> p = [p1 p2]$
- $>> k = place(G, H, p)$

It is possible to adjust the steady-state error of the system by means of a gain $k0$. Calculate the gain required for the system output to stabilize around the reference signal. Include the complete calculation of this gain in the final report. Once calculated, modify the gain $k0$ in the system and verify that the error has been adjusted.

To check the results obtained in this section, complete the file $P3$.mlx and run it using the "Run" button. To do this, first set the current Matlab directory to the folder of the practice session (see Figure 1) and then open the file $P3$.mlx from the current directory.

P3-State Feedback Control

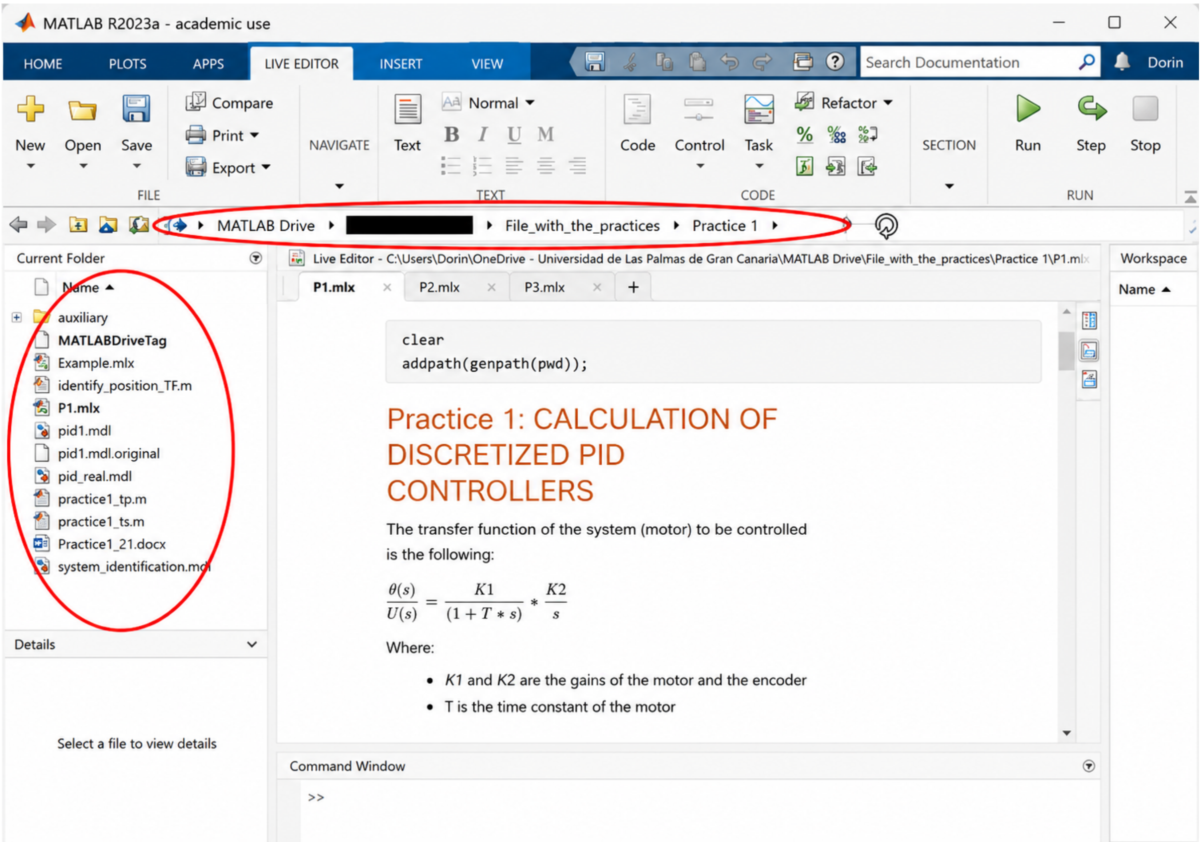

Figure 1: Matlab interface

1. Run the Simulink tool and open the file "*realiment_estado.mdl*". This file simulates the motor scheme taking into account the selected state variables, as can be seen in the following figure.

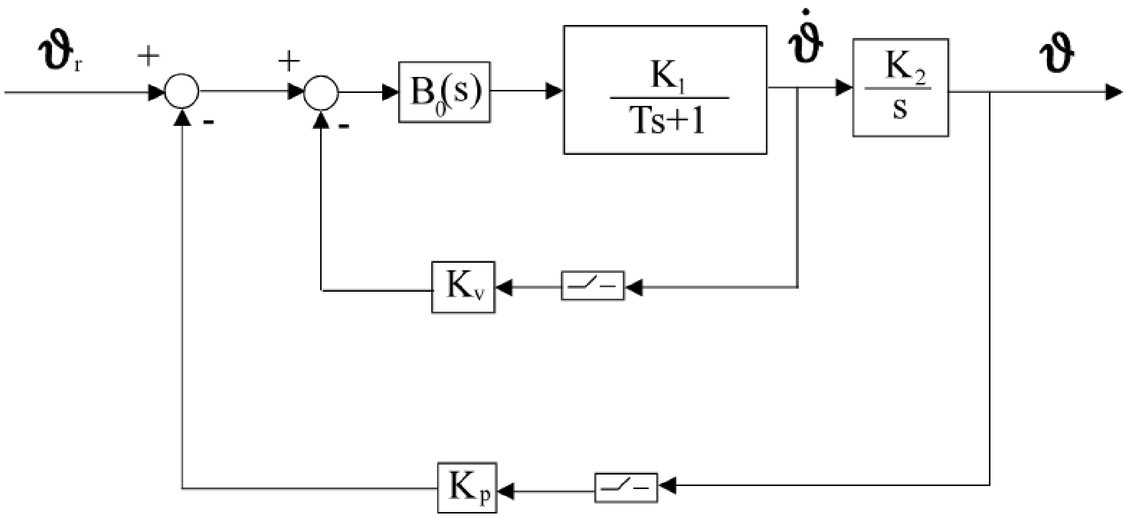

Figure 2: Motor feedback scheme

2. Place the calculated gains in the corresponding blocks and simulate.

3. Analyze the results obtained.
   - Measure the peak time and overshoot of the system output, as well as the number of samples until settling.
   - What is the value of the output once it has stabilized? What is the steady-state error?