Control Engineering II

Exam

C.A. Monje, D. Copaci, M. Malfaz, J. Muñoz,
L. Mena, L. Moreno, S. Garrido

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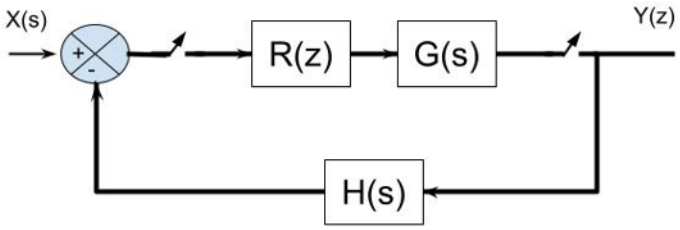
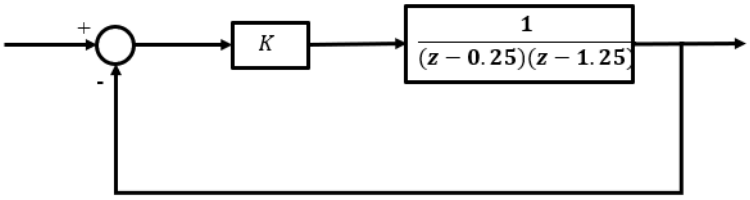
Exam

- **Exercise 1**
  Find the equivalent discrete transfer function of the following diagram using the residue method:

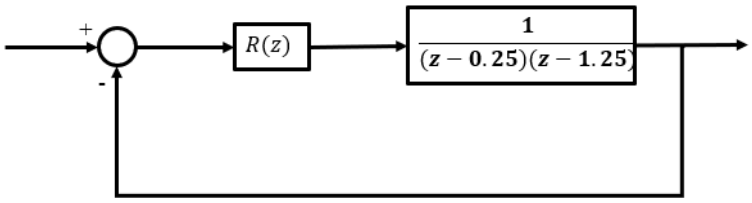


  where:
  $R(z) = \frac{z-1}{z}$
  $G(s) = \frac{1-e^{-Ts}}{s}\frac{5}{s+1}$
  $H(s) = \frac{1}{s}$

- **Exercise 2**
  Consider the following system:



  - Using the Jury test, determine the values of K that make the system stable.
  - Now consider:



    Determine the simplest possible controller, using the root-locus design method, such that the closed-loop system has dominant poles at $z_d = 0.25 \pm i0.25$.
  - For this controller, determine the position error.

  Notes:
  - *Draw and derive the root locus in detail.*
  - *Explain and justify the entire procedure.*

- **Exercise 3**
  Consider the following discrete-time system:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(k)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

  - Determine the output controllability matrix. Is this system completely output controllable?
  - If the initial state is $x(0) = (0\ 0)^t$, what input sequence $u(0), u(1)$ is required for the system output to reach $y(2) = (3\ 9)^t$?
  - The system now has a single output, and therefore the output expression for the same system becomes:

$$y(k) = [3\ 1] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

    The system is controlled using state feedback. Determine the state-feedback matrix K that places the closed-loop system poles at $p_{1,2} = 0,5 \pm j0,5$.

- **Exercise 4**
  Given the system:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.1 & 0.5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k) \tag{1}$$

$$y(k) = \begin{bmatrix} 0.5 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \tag{2}$$

  - Determine the state observability of the system.
  - Given the output sequence $y(0) = 1$, $y(1) = 2$, for the inputs $u(0) = 0$, $u(1) = 0$, determine the initial state of the system $x(0)$.
  - Given an observer gain $K_e = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ for a full-order state observer, determine the observer poles and the system poles. Is the observer gain matrix suitable for estimating the state of the system?